UNIT II One Dimensional Problems

The structure shown in figure.1, is subjected to an increase in temperature of $80°C$. Determine the displacements, stress and support reactions. Assume the following data:

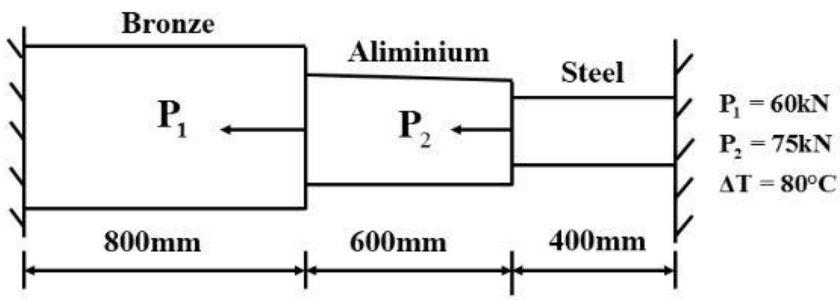
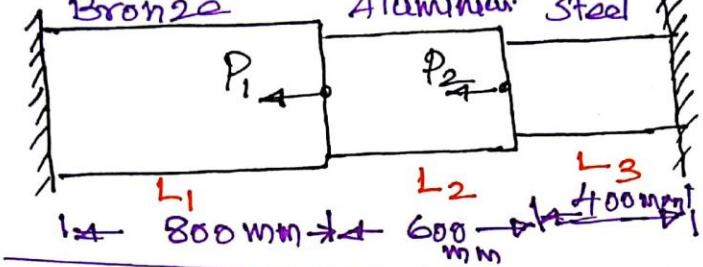


Figure. 1.

Bronze	Aluminium	steel
$A=2400mm^2$	$1200mm^2$	$600mm^2$
E=83GPa	E=70GPa	E=200GPa
$\alpha = 18.9\times10^{-6}\ /\ °C$	$\alpha = 23\times10^{-6}\ /\ °C$	$\alpha = 11.7\times10^{-6}\ /\ °C$

The structure shown in figure is subjected to an increase in temperature of 80°c. Determine the displacements, stresses and support reactions. Assume the following data.



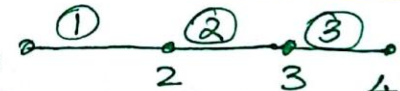
$P_1 = 60kN$

$P_2 = 75 kN$

$\Delta T = 80^\circ C$

Bronze	Aluminium	Steel
$A_1 = 2400 mm^2$	$A_2 = 1200 mm^2$	$A_3 = 600 mm^2$
$E_1 = 83 GPa$	$E_2 = 70 GPa$	$E_3 = 200 GPa$
$\alpha_1 = 18.9\times10^{-6}/^\circ c$	$\alpha_2 = 23\times10^{-6}/^\circ c$	$\alpha_3 = 11.7\times10^{-6}/^\circ c$

Solution: FEA Model



Finite element equation for two noded bar element for one dimensional is given by,

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \Rightarrow [K]\{U\} = \{F\}$$

Stiffness matrix

→ Element ①

$$K^{(1)} = \frac{2400\times83\times10^3}{800}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 249\times10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 249 & -249 \\ -249 & 249 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

Element ②

$$K^{(2)} = \frac{1200\times70\times10^3}{600}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 140\times10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 140 & -140 \\ -140 & 140 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

Element ③

$$K^{(3)} = \frac{600\times200\times10^3}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \end{array}$$

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Global matrix $[K] = K^{(1)} + K^{(2)} + K^{(3)}$

$$
10^3 \begin{array}{cc}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \\
\begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 249+140 & -140 & 0 \\ 0 & -140 & 140+300 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}
\end{array}
$$

Displacement Vector

$$U = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

Load Vector $\{F\} = EA\alpha\Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

Element$^{(1)}$ $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 83 \times 10^3 \times 2400 \times 18.9 \times 10^{-6} \times 80 \times \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 10^3 \begin{Bmatrix} -301.1904 \\ 301.1904 \end{Bmatrix} \begin{array}{c} 1 \\ 2 \end{array}$$

Element$^{(2)}$ $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 70 \times 10^3 \times 1200 \times 23 \times 10^{-6} \times 80 \times \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 10^3 \begin{Bmatrix} -154.56 \\ 154.56 \end{Bmatrix} \begin{array}{c} 2 \\ 3 \end{array}$$

Element$^{(3)}$ $\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = 200 \times 10^3 \times 600 \times 11.7 \times 10^{-6} \times 80 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 10^3 \begin{Bmatrix} -112.32 \\ 112.32 \end{Bmatrix}$$

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Global force Vector

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = 10^3 \times \begin{Bmatrix} -301.1904 \\ 301.1904 - 154.56 \\ 154.56 - 112.32 \\ 112.32 \end{Bmatrix}$$

$$= 10^3 \times \begin{Bmatrix} -301.1904 \\ 146.6304 \\ 42.24 \\ 112.32 \end{Bmatrix} * 10^3 \times \begin{bmatrix} -301.1904 \\ 146.6304 - 60 \\ 42.24 - 75 \\ 112.32 \end{bmatrix}$$

* load is acting towards left $P_1 = -60 \times 10^3$ $P_2 = -75 \times 10^3$

Element Equation.

$$\cancel{10^3} \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \cancel{10^3} \times \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

Apply the boundary condition

$u_1 = 0$, $u_4 = 0$

$$\begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{bmatrix} = \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

In above equation, $u_1 = 0$, so, neglect first row and first column of [K] matrix, $U_4 = 0$ So, neglect fourth row and fourth column of [K] matrix. Hence the equation reduces to

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$$\begin{bmatrix} 389 & -140 \\ -140 & 440 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 86.6304 \\ -32.76 \end{bmatrix}$$

$$389 u_2 - 140 u_3 = 86.6304$$

$$-140 u_2 + 440 u_3 = -32.76$$

Solving

$u_2 = 0.2212$ mm $\quad u_3 = -0.00345$ mm

Thermal stress $\sigma = E \frac{du}{dx} - E\alpha\Delta T$

For element(1) $\sigma_{(1)} = \frac{E_1 (u_2 - u_1)}{l_1} - E_1 \alpha_1 \Delta T$

$$= \frac{83 \times 10^3 [0.2212 - 0]}{800} - 83 \times 10^3 \times 18.9 \times 10^{-6} \times 80$$

$= -102.5455$ N/mm^2 [Compressive Stress]

For element(2) $\sigma_{(2)} = \frac{70 \times 10^3 [-0.00345 - 0.2212]}{600} - 70 \times 10^3 \times 23 \times 10^{-6} \times 80$

$\sigma_2 = -155.009$ N/mm^2 [Compressive Stress]

For Element(3)

$$\sigma_{(3)} = \frac{200 \times 10^3 [0 + 0.00345]}{400} - 200 \times 10^3 \times 11.7 \times 10^{-6} \times 80$$

$\sigma_{(3)} = -185.475$ N/mm^2 [Compressive Stress]

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Reaction force

$$\{R\} = [K]\{u^*\} - \{F\}$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = 10^3 \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2212 \\ -0.00345 \\ 0 \end{bmatrix} - 10^3 \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

$$= 10^3 \begin{bmatrix} -55.0788 \\ 86.5018 \\ -32.486 \\ 1.035 \end{bmatrix} - 10^3 \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

$$= 10^3 \times \begin{bmatrix} 246.1116 \\ 0 \\ 0 \\ -113.35 \end{bmatrix}$$

Result.

Displacement

$u_1 = 0$

$u_2 = 0.2212\,mm$

$u_3 = -0.00345\,mm$

$u_4 = 0$

Stress

$\sigma_1 = -102.5465$

$\sigma_2 = -155.009\ N/mm^2$

$\sigma_3 = -18\cancel{8}.475\ N/mm^2$

Reaction force

$R_1 = 246.11\cancel{66} \times 10^3\ N$

$R_2 = -113.35 \times 10^3\ N$

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