23MCT203 - Theory of Control Engineering

Basic Concepts of State Space Model

The following basic terminology involved in this chapter.

State

It is a group of variables, which summarizes the history of the system in order to predict the future values (outputs).

State Variable

The number of the state variables required is equal to the number of the storage elements present in the system.

Examples − current flowing through inductor, voltage across capacitor

State Vector

It is a vector, which contains the state variables as elements.

In the earlier chapters, we have discussed two mathematical models of the control systems. Those are the differential equation model and the transfer function model. The state space model can be obtained from any one of these two mathematical models. Let us now discuss these two methods one by one.

Definition of State-Space Models

State-space models are models that use state variables to describe a system by a set of first-order differential or difference equations, rather than by one or more *n*th-order differential or difference equations. If the set of first-order differential equation is linear in the state and input variables, the model is referred to as a *linear* state space model.

Note

Generally, the System Identification Toolbox™ documentation refers to linear state space models simply as state-space models. You can also identify nonlinear state space models using grey-box and neural state-space objects. For more information, see Available Nonlinear Models.

The linear state-space model structure is a good choice for quick estimation because it requires you to specify only one parameter, the *model order*, n. The *model order* is an integer equal to the dimension of *x(t)* and relates to, but is not necessarily equal to, the number of delayed inputs and outputs used in the corresponding difference equation. State variables *x(t)* can be reconstructed from the measured input-output data, but are not themselves measured during an experiment.

Continuous-Time Representation

It is often easier to define a parameterized state-space model in continuous time because physical laws are most often described in terms of differential equations. In continuous-time, the linear state-space description has the following form:

$$\dot{x}(t)=Fx(t)+Gu(t)+\tilde{K}w(t) \quad y(t)=Hx(t)+Du(t)+w(t) \quad x(0)=x0$$

The matrices F, G, H, and D contain elements with physical significance—for example, material constants. *x0* specifies the initial states.

Note

$\tilde{K} = 0$ gives the state-space representation of an Output-Error model. For more information, see What Are Polynomial Models?.

You can estimate continuous-time state-space model using both time- and frequency-domain data.

Discrete-Time Representation

The discrete-time linear state-space model structure is often written in the *innovations form* that describes noise:

$$x(kT+T)=Ax(kT)+Bu(kT)+Ke(kT) \quad y(kT)=Cx(kT)+Du(kT)+e(kT) \quad x(0)=x0$$

where T is the sample time, *u(kT)* is the input at time instant kT, and *y(kT)* is the output at time instant kT.

Note

$K=0$ gives the state-space representation of an Output-Error model. For more information about Output-Error models, see What Are Polynomial Models?.

Discrete-time state-space models provide the same type of linear difference relationship between the inputs and outputs as the linear ARMAX model, but are rearranged such that there is only one delay in the expressions.

You cannot estimate a discrete-time state-space model using continuous-time frequency-domain data.

The innovations form uses a single source of noise, *e(kT)*, rather than independent process and measurement noise. If you have prior knowledge about the process and measurement noise, you can use linear grey-box estimation to identify a state-space model with structured independent noise sources. For more information, see Identifying State-Space Models with Separate Process and Measurement Noise Descriptions.