23MCT203 - Theory of Control Engineering

Linearity versus nonlinearity

Linear Control System

When we talk about a linear control system it is all about how it responds to inputs. Picture it this way imagine you have a magical machine called 'System S' that does stuff based on what you give it. Now this machine follows two important rules:

- **Homogeneity:** If you give it something to do say a task or an instruction & you make that instruction bigger or smaller by a certain amount the result from the machine gets bigger or smaller by the same amount. It is like when you increase the work you ask it to do the result also increases by the same factor.
- **Additivity:** Now, if you give this machine two different tasks separately, it gives you two results, right?. Now, here is the interesting part. If you combine these two tasks and give them to the machine as one big task (adding the two smaller tasks together), the result you get is the same as if you added the results of the two smaller tasks separately. It is like the machine does not care if you give it two jobs or one big combined job, the result is the sum of the individual results.

When we talk about linear control systems, we are talking about systems that play by these rules. They were the types of systems that follow these two principles that is they respond predictably when you make the task bigger or smaller & their response to combined tasks is just the sum of their responses to the individual tasks.

Example of Linear Control System

Think of a simple circuit with just resistors and a battery that gives a steady flow of electricity. In this circuit, if you change the voltage from the battery that's make it stronger or weaker the resulting current in the circuit changes by the same factor. That's the homogeneity part – change one thing & the other changes in sync. Now if you have a bunch of resistors connected in different ways and you measure the voltage and current at different points you will find that if you sum up the voltages or currents separately it's the same as if you add them all together at once. In other words the total effect is just the sum of all the individual effects. This behavior makes it a linear control system.

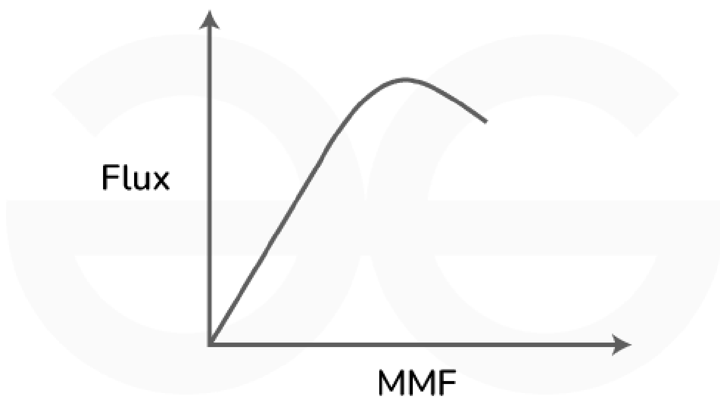
So, this circuit when we ignore any funky side effects and assume everything working perfectly shows a linear relationship between voltage and current. It is a good example of a linear control system because it plays by the rules of homogeneity and additivity.

Non-Linear Control System

Nonlinear control system is basically one that does not follow the simple rules we talked about earlier. Remember how we said in linear systems changing one thing changes another thing by the same amount? Well, in real life, most systems don't work like that. They are a bit more complicated.

Take something like a DC machine's magnetization curve which shows how the magnetic field inside the machine changes with the electricity going through it. At first when you increase the electricity accordingly the magnetic field inside also increases in a kind of straight line. That is the linear part, where things change in a predictable way. But after a point something interesting happens the system starts to act differently. Even if you keep increasing the electricity the magnetic field does not change in the same straight-line way anymore. It reaches a point where it is kind of maxed out, like a sponge that cannot soak up any more water. That's what we call saturation & it makes the relationship between electricity and the magnetic field non-linear.

So, nonlinear systems are the ones that don't stick to simple, predictable patterns throughout their whole operation. They might start off following a rule, but at some point, they veer off and behave in unexpected ways, like this magnetization curve in a DC machine.



<u>Graph of DC Machine</u>

<u>Example of Non-linear system</u>

Beyond a certain point of increasing electricity input the magnetic field inside the machine does not increase at the same rate. Instead, it starts reaching a point where it becomes saturated. The main characteristic of this is a saturation represents a nonlinear behavior where additional increases in electricity no longer produce equivalent increases in the magnetic field strength. It reaches a maximum limit.

Applications of Linear and Non-Linear Control System

<u>Linear Control System</u>

- Linear Algebra in Control Systems: Eigenvalue and eigenvector and their applications are used in a linear control system.
- Stability Analysis: Criteria for stability such that eigenvalues and stability regions in linear systems.
- Transfer Functions and Frequency Domain Analysis: Understanding system behavior using transfer functions, frequency response & Bode plots.

<u>Non-Linear Control System</u>

- Nonlinear Differential Equations: Methods to solve and understand the systems governed by nonlinear differential equations.
- Nonlinear Control Strategies: Developing control techniques specifically tailored for nonlinear systems, such as sliding mode control, adaptive control, and back stepping control.
- Nonlinear Stability Analysis: Evaluating stability in systems that does not adhere to linear stability concepts using Lyapunov stability theory & other nonlinear stability analysis methods.

<u>Difference Between Linear and Non-linear Control System</u>

Linear Control System	Non-linear Control System

Linear Control System	Non-linear Control System
Linear control system responds predictably to change in inputs.	It does not respond predictably to changes in input beyond a point.
Changes in input scale linearly affect the output.	Does not exhibit linear scalability with inputs.
Predict the behavior within the specified limits.	Unpredictable behavior of nonlinear control system especially beyond certain thresholds.
This system follows the specific rules consistently.	Does not follow simple predictable patterns throughout operation.

Block Diagram