COURTESY
https://nptel.ac.in/courses/112104117/ui/Course_home-8.htm

Problem 1. A hydro-turbine is required to give 25 mW at 50 m heat and 90 r.p.m. runner speed. The laboratory facilities available permit testing of 20 kW model at 5m head. What should be the model runner speed and model prototype scale ratio?

Solution: P_1 = 25 mW $\quad$ P_2 =20 kW

N_1 = 90 r.p.m. $\quad$ H_2 =5 m

H_1 =50m

Scale ratio $= \frac{D_1}{D_2} = \sqrt{\frac{P_1}{P_2}}\left(\frac{H_2}{H_1}\right)^{3/4} = \sqrt{\frac{25\times10^6}{20\times10^3}}\times\left(\frac{5}{50}\right)^{3/4}$ =6.29

$N_2 = N_1 \times \frac{D_1}{D_2}\sqrt{\frac{H_2}{H_1}}$ =90x6.29x $\sqrt{\frac{5}{50}}$ =179r.p.m.

Problem 2. In an inward flow reaction turbine having vertical shaft, water enters the runner from the guide blades at an angle of 155° with the runner blade angle at entry being 100°. Both these angles are measure from the tangent at runner periphery drawn in the direction of runner rotation. The flow velocity through the runner is constant, water enters the draft tube from tile runner without whirl and the discharge from the draft tube into the tail race takes place with a velocity of 2.5 m/s. The runner has the dimensions of 40 cm external diameter and 3.8 cm inlet width. The turbine works with a net head of 35m and the loss of head in the turbine due to fluid n is 4m of water. Draw vector diagrams and calculate:

1. Speed of the runner

2. Runner blade angle at a point on the outlet edge where the radius of rotation is 9 cm.

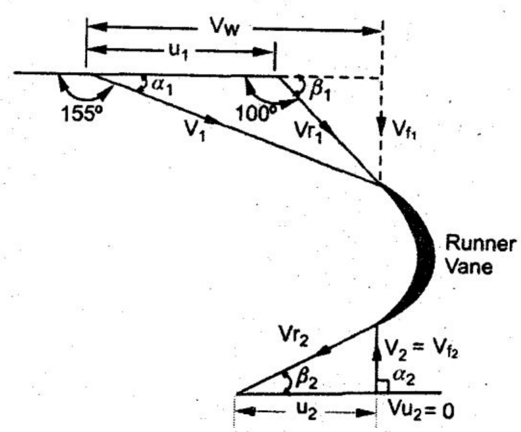
3. Power generated by the turbine and its specific speed.

4. Inlet diameter of the draft tube.

Solution. Velocities at inlet and exit are related by the expression:

$$\frac{v_{r2}^2 - v_{r1}^2}{2g} = \frac{u_2^2 - u_1^2}{2g}, \frac{v_{r2}^2}{2g} = \frac{v_{r1}^2}{2g} + \frac{u_2^2 - u_1^2}{2g}$$

From the inlet velocity triangle



α_1 = (180—155) =25°

β_1 = (180 — 100) = 800

$$v_{u1} = \frac{v_{f1}}{\tan\alpha_1}$$

$$= \frac{v_{f1}}{\tan 25^\circ} = 2.144\, v_{f1}$$

$$u_1 = v_{u1} - \frac{v_{f1}}{\tan\beta_1}$$

$$= 2.144\, v_{f1} - \frac{v_{f1}}{\tan 80°} = 1.968\, v_{f1}$$

Since the discharge is in radial direction, $v_{u1} = 0$

Work done = $\frac{v_{u1} u_1}{g} = \frac{2.144 v_{f1} \times 1.968 v_{f1}}{9.81}$ = 0.43 v_{f1}^2

From the energy balance,

Head supplied

= (work done) + (kinetic heat at exit) + (losses in the runner)

$$3.5 = 0.43\, v_{f1}^2 + \frac{2.5^2}{2 \times 9.81} + 4$$

v_{f1} = 8.45m/s

u_1 = 1.968 v_{f1} = 1.968 x 8.45 = l6.63m/s

1. $u_1 = \frac{\pi d_1 N}{60}$;1663= $\frac{\pi \times 0.4 \times N}{60}$

N = $\frac{16.63 \times 60}{\pi \times 0.4}$ = 794 r.p.m.

2. From outlet velocity triangle:

$v_2 = v_{f2} = v_{f1}$ =8.45m/s

Peripheral velocity of the outer edge at 9 cm radius

$u_2 = u_1 \times \frac{r_2}{r_1}$ = 16.63 x $\frac{9}{20}$ = 7.48 m/s

$\tan\beta_2 = \frac{v_{f2}}{u_2} = \frac{8.45}{7.48}$ = 1.13 ; vane angle at outlet, β_2 = 48.5°

Discharge through the turbine, Q $= \pi d_1 b_1 v_{f1} = \pi$ x 0.4 x 0.038 x 8.45

= 0.4035 m^3/s

3. Power developed by the turbine,

$$p = w Q \frac{v_{u1} . u_1}{g} = w Q (0.43 \, v_f^2)$$

= 9810 x 0.4035 x 0.43 $(8.45)^2$ 121.5 x 10^3 W = 121.5 kW

Assume a mechanical efficiency of 98%

4. Power available at turbine shaft = 121. 5 x 0.98 = 119.07

Specific speed of the turbine, Ns $= \frac{N\sqrt{p}}{H^{5/4}} = \frac{794 \times \sqrt{119.07}}{35^{5/4}}$ = 101.77

5. Inlet area of draft tube = $\frac{\text{discharge}}{\text{flow velocity}} = \frac{0.4035}{8.45}$ = 0.04775 m^2

If d is the inlet diameter of the tube,

$$\frac{\pi}{4} d^2 = 0.04775$$

$$d = \left(\frac{0.04775 \times 4}{\pi}\right)^{1/2} = 0.246 m$$

r

Problem.3. The velocity of whirl at inlet to the runner of an inward flow reaction turbine is ($3.15\sqrt{H}$) m/s and the velocity of flow at inlet is ($1.05\sqrt{H}$) m/s. The velocity of whirl at exit is ($0.22\sqrt{H}$) m/s in the same direction as at inlet and the velocity of flow at exit is ($0.83\sqrt{H}$) where H is the head in meters. The inner diameter of the runner is 0.6 times the outer diameter. Assuming hydraulic efficiency of 80%, compute the angles of the runner vanes at inlet and exit.

Solution.

$$\eta_h = \frac{(V_w u - V_{w1} u_1)}{gH}; \left(\frac{u}{D}\right) = \left(\frac{u_1}{D_1}\right)$$

$$0.8 = \frac{[(3.15\sqrt{H})u - (0.22\sqrt{H})\,0.6u]}{9.81H}$$

$$u = (2.60\sqrt{H}) \text{ and } u_1 = (1.56\sqrt{H})$$

From inlet velocity triangle, we have

$$\tan\theta = \frac{V_f}{(V_w - u)} = \frac{1.05\sqrt{H}}{(3.15\sqrt{H}) - (2.60\sqrt{H})}$$

1.9091

θ = 62°21’

From outlet velocity triangle, we have

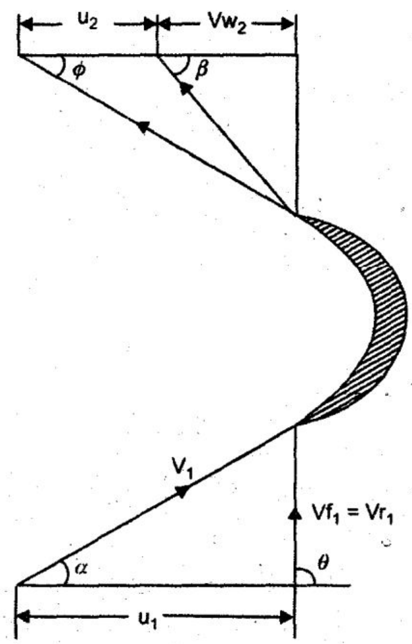
$$\tan\phi = \frac{V_{f1}}{(u_1 - V_{w1})} = \frac{0.83\sqrt{H}}{(1.56\sqrt{H}) - (0.22\sqrt{H})}$$

=0.6194

ϕ =31°46’

Problem.4. The inlet and the outlet runner blade angles of a propeller turbine are $90°$ and 25° respectively to the tangential direction of the runner. The inlet guide vane angle is 30°. The speed of the turbine 30 rpm. The mean diameter of the runner blades is 3.6 m and the area of flow is 30 m^2. Assuming that the velocity of flow is constant throughout, determine (1) Discharge (ii) Power developed (iii) Hydraulic efficiency (iv) Specific speed.

Solution.



D_0 =3.6m

N =30 r.p.m.

θ **= 90°**

ϕ = 25°

a = 30°

Flow area, a = $30\ m^2$

Runner blade angle at inlet is radial

$V_{r1} = V_{f1}$ and $u_1 = Vw_1$

As velocity of flow is constant so. $V_{f1} = V_{f2}$

$$u_1 = \frac{\pi D_0 N}{60} = \frac{\pi \times 3.6 \times 30}{60}$$

= 5.65 m/s

Also $u_2 = u_1$

= 5.65 m/s

From inlet velocity triangle

$$\tan \alpha = \frac{V_{f1}}{u_1}$$

$$\tan 30^\circ = \frac{V_{f1}}{5.65}$$

V_{f1} =5.65xtan30°

=3.262m /s

$V_{w1} = u_1$ =5.65m/s

From outlet velocity triangle, $\tan \phi = \frac{V_{f2}}{u_2 + V_{w2}}$

$$\tan 25^\circ = \frac{3.262}{5.65 + V_{w2}}$$

$V_{w2} + 5.65 = \frac{3.262}{\tan 25}$ =7 m/s

V_{w2} = 7 - 5.65 = 1.35 m/s

$V_2 = \sqrt{V_{f2}^2 + V_{w2}^2}$ =3.529 m/s

We have, $H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w1} \cdot u_1 - V_{w2} \cdot u_2]$

$H - \frac{(3.529)^2}{2 \times 9.81} = \frac{1}{9.81}$ [5.65 x 5.65 — 1.35 x 5.65]

$H = \frac{31.92 - 7.62}{9.81} + \frac{12.45}{19.62}$

H = 2.47 + 0.634 3.104 m

(1)Hydraulic efficiency is given by

$\eta_h = \frac{V_{w1} \cdot u_1 - V_{w2} \cdot u_2}{g \times H} = \frac{31.92 - 7.62}{9.81 \times 3.104}$

=0.798 = 79.8%.

(2)Discharge through turbine, Q = Area of flow x Velocity of flow

=30x3262=97.86 m^3/s

(3)Power developed by turbine $= \frac{\text{work done per second}}{1000}$

$= \frac{1}{g} \frac{[V_{w1} \cdot u_1 - V_{w2} \cdot u_2]}{1000}$ × Weight of water

$$= \frac{1}{9.81}\frac{[31.92-7.62]}{1000} \times 1000 \times 9.81 \times 97.86$$

= 2378 kW

(4)Specific speed is given by $N_s = \frac{N\sqrt{p}}{H^{5/4}} = \frac{30 \times \sqrt{2378}}{(3.104)^{5/4}} = \frac{1462.94}{4.12} =$ 355.08 rpm

---

Problem.5. In a Francis turbine of very low specific speed, the velocity of flow from inlet to exit of the runner remains constant. If the turbine discharges radially, show that the degree of reaction p can be expressed as

$$\rho = \frac{1}{2} - \frac{1}{2}\left[\frac{\cot\theta}{(\cot\alpha - \cot\theta)}\right]$$

where a and θ are the guide and runner vane angles respectively and the degree of reaction p is equal to the ratio of pressure drop to the hydraulic work done in the runner, assuming that the losses in the runner are negligible.

Solution. Applying Bernoulli's equation between the inlet and exit of the runner and neglecting the potential difference, we get

$$\frac{p}{w} + \frac{V^2}{2g} = \frac{P_1}{w} + \frac{V_1^2}{2g} + \frac{V_w u}{g}$$ (for radial discharge)

Where $\frac{p}{w}$ and $\frac{p_1}{w}$ are the pressure heads at the inlet aid the exit of the runner respectively.

Thus pressure head drop due to hydraulic work done in the runner is given by

$$\frac{p}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{V^2}{2g} + \frac{V_w u}{g}$$

Now

$$\rho = \frac{\left(\frac{p}{w} - \frac{p_1}{w}\right)}{\frac{V_w u}{g}}$$

Or

$$\rho = \frac{\frac{V_1^2}{2g} - \frac{V^2}{2g} + \frac{V_w u}{g}}{\frac{V_w u}{g}}$$

Or

$$\rho = 1 + 1 + \frac{1}{2}\left[\frac{V_1^2 - V^2}{V_w u}\right] \quad \ldots (1)$$

For radical discharge

$$V_1 = V_{f1} = V_f$$

Also

$$V_w = V \cos\alpha$$

$$V_f = \sin\alpha$$

$$u = V_w - V_f \cot\theta$$

Or u =V [cos a-sin a cot θ]

And

$$V_1 = V_f = V \sin\alpha$$

Thus, introducing these values in equation (i) above and simplifying it, we get.

$$\rho = \frac{1}{2} - \frac{1}{2}\left[\frac{\cot\theta}{\cot\alpha - \cot\theta}\right]$$

Problem.6. An inward flow pressure turbine has runner vanes which are radial-at the inlet and inclined backward at 45° to the tangent at discharge. The guide vanes are inclined at 15° to tangent at inlet and velocity of water leaving the guides in 24 m/sec. Determine correct speed for runner and absolute velocity of water at point of discharge if diameter at entry is twice that at discharge and width at entry is 0.6 times that at discharge.

Solution.

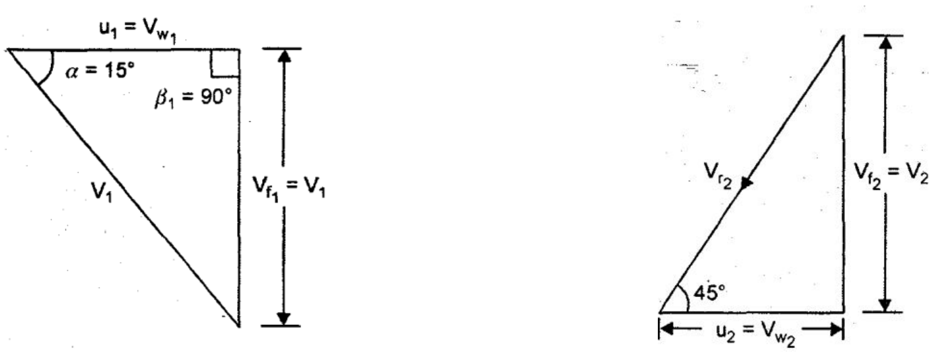


Fig. Input and outlet velocity triangle

Given α_1 **=15°** $\beta_2 = 45°$

Vr_2 = 24 m/s $b_1 = 0.6\,b_2$

$d_1 = 2d_2$

In outlet velocity triangle

$$Vf_2 = V_2 = \frac{Vr_2}{\sin\beta} = \frac{24}{0.707}$$ = m/s Ans.

We know

$$Q = \pi\, d_1 b_1\, Vf_1 = \pi\, d_2\, b_2\, Vf_2$$

$$Vf_1 = \frac{d_2}{d_1} \times \frac{b_2}{b_1} \times Vf_2$$

$$= \frac{d_2}{2d_2} \times \frac{b_2}{0.6\, b_2} \text{ x}33.94$$

$$= \frac{10}{12} \text{ x } 33.94 = 28.28 \text{ m/s}$$

$$\tan \alpha_1 = \frac{Vf_1}{u_1}$$ (For radial discharge)

$$u_1 = \frac{Vf_1}{\tan \alpha_1} = \frac{28.28}{0.267} = 105.94 \text{ m/s Ans.}$$