



UNIT - I

Logics and Proofs

Propositions: Defn: A Proposition is a declarative sentence that is either true or false, but not both.

eg: Chennai is the capital of Tamil Nadu (True);
 $1+2=6$ (False); $2+7=9$; (True); Delhi is in America (False)

Non-Propositions: Defn: Questions, exclamations and commands are non-proposition.

eg: This statement is false (we can't say T or F)

Do you speak English? (Question)

Obeey my orders (Command); $x+5=3$ (neither nor F)

Notations: P, Q, R, S \rightarrow propositions; T \rightarrow True; F \rightarrow False;

Types of Propositions: (i) Primary / Atomic / Simple Propositions:

Declarative sentences which cannot be further split into simpler sentences are called Atomic Statements / P / Simple propositions. eg: Rama is a boy; Rose is a beautiful flower.

(ii) Compound / Molecular / Composite Propositions:

Statements which contain one or more primary ^{statements} and some connectives are called compound / Mo / Com / propositions.

eg: In spite of Ram's Busy Schedule, he attended the exam.

Lotus is a pink flower and also it is the National Flower of India.

Defn: Connectives: Connective is an operation which is used to connect / compare two or more statements.

FIVE BASIC CONNECTIVES

English Language Usages	Logical Connectives	Type of operator	Symbols
① and	Conjunction	binary	\wedge
② or	disjunction	binary	\vee
③ not	negation (or) denial	Unary	\neg or \sim
④ if... then	implication (or) conditional	binary	\rightarrow
⑤ if and only if	biconditional	binary	\leftrightarrow



Defn: Truth Table.

A table, giving the truth values of a compound statement in terms of its component parts, is called a Truth Table.

Truth Table

P	Q	$P \wedge Q$	$P \vee Q$	$\neg P$	$P \rightarrow Q$	$P \leftrightarrow Q$	Converse $Q \rightarrow P$	Inverse $\neg P \rightarrow \neg Q$	Contradiction $TQ \rightarrow$
T	T	T	T	F	T	T	T	T	T
T	F	F	T	F	F	F	T	T	F
F	T	F	T	T	T	F	F	F	T
F	F	F	F	T	T	T	T	T	T

Examples:

1) Conjunction [\wedge] AND

(i) P: It is snowing [True] Q: I am cold [True]

$P \wedge Q$: It is snowing and I am cold [True]

(ii) P: $2 < 6$ [True] Q: $2 + 6 = 9$ [False]

$P \wedge Q$: $2 < 6$ and $2 + 6 = 9$ [False]

2) Disjunction [\vee] OR

(i) P: 2 is a +ve integer [True]

Q: $\sqrt{2}$ is a rational number [True]

$P \vee Q$: 2 is a +ve integer or $\sqrt{2}$ is a rational number [True]

(ii) P: $3 + 4 = 6$ [False]

Q: London is the capital of India [False]

$P \vee Q$: $3 + 4 = 6$ or London is the cap. of India [False]

(iii) P: $2 + 3 = 5$ [True]; Q: $3 < 2$ [False]

$P \vee Q$: $2 + 3 = 5$ (or) $3 < 2$ [True]

3) Negation [\neg] NOT

(i) P: Today is Monday [True]; $\neg P$: Today is not Monday [False]

(ii) P: $x < y$ [True]; $\neg P$: $x \neq y$ or $x \geq y$ [False]



Conditional : [if... then] [\rightarrow]



(i) $P : 2 + 5 = 8$ [False] ; $Q = 7 + 3 = 9$ [False]

$P \rightarrow Q$: If $2 + 5 = 8$, then $7 + 3 = 9$ [True]

(ii) P : The sun is shining today ; $Q : 2 + 8 = 6$ [False]

$P \rightarrow Q$: If the sun is shining today, then $2 + 8 = 6$ [False]

5) Biconditional [if and only if] [\leftrightarrow] [equivalence]

(i) P : You cannot take the flight [False]

Q : You do not buy a ticket [False]

$P \leftrightarrow Q$: You cannot take the flight if and only if you do not buy a ticket [True]

(ii) $P : 5 < 6$ [True] ; $Q : 7 > 8$ [False]

$P \leftrightarrow Q$: $5 < 6$ if and only if $7 > 8$ [False]

Correct form

$(\neg P \wedge Q) \mid \neg(P \wedge Q)$

$(P \rightarrow Q) \rightarrow (P \wedge Q)$

$(P \rightarrow Q)$

$((P \wedge Q) \rightarrow Q)$

[well-formed formulae]

Incorrect Form

$\neg P \wedge Q$

$(P \rightarrow Q) \rightarrow (\wedge Q)$

$(P \rightarrow Q)$

$(P \wedge Q) \rightarrow Q$

Examples : i) How can this English sentence be translated into a logical expression? "You can access the Internet from campus only if you are a computer science major or you are not a freshman".

Ans: A : You can access the Internet from campus.

B : You are a computer science major

C : You are a freshman.

$\neg C$: You are not a freshman.

$B \vee \neg C$: You are a comp. S. major / You are not freshman

$A \rightarrow (B \vee \neg C) : Q_n$



Write the following in symbolic form.

If either Ram takes Calculus or Krishna takes Sociology then Sita will take English.

Ans: A = Ram takes calculus B = Krishna takes Sociology
C = Sita takes English.

$$(A \vee B) \rightarrow C$$

3) State the Truth Value of "If tigers have wings then the earth travels round the sun".

Ans: Let P: Tigers have wings [False]
Q: The earth travels round the sun [True]
∴ The given statement $P \rightarrow Q$, has the truth value "True".

4) Construct the truth table for (i) $\neg P$

(i) 1 variable $\rightarrow 2^1$ values

P	$\neg P$	$\neg \neg P$
T	F	T
F	T	F

(ii) $\neg P \wedge Q$

(iii) $P \wedge (P \vee Q)$

(iv) $(P \rightarrow Q) \wedge (Q \rightarrow P)$

(ii) 2 variables $\rightarrow 2^2$ values

P	Q	$\neg P$	$\neg P \wedge Q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

(iv) P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

(iii) 2 variables $\rightarrow 2^3$ values

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T
T	F
F	F
T	T



(i) $(\neg P \vee \neg Q) \rightarrow P$ (ii) $(Q \wedge (P \rightarrow Q)) \rightarrow P$

(iii) $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$

(iii)	P	Q	$\neg P$	$P \rightarrow Q$	$\neg P \vee Q$	$(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
	T	T	F	T	T	T
	T	F	F	F	F	T
	F	T	T	T	T	T
	F	F	T	T	T	T

H.W) Using the statements P: x is rich Q: x is happy write the statements in symbolic form:

- a) x is poor b) x is poor but happy c) x is rich or unhappy
 - d) x is neither rich nor happy e) x is poor or he is both rich & unhappy
- Ans: $\neg A$; $\neg A \wedge B$; $\neg A \vee \neg B$; $\neg A \wedge \neg B$; $\neg A \vee (A \wedge \neg B)$

2) Write the statements for the symbolic form.

P: It is a hot day; Q: Temperature is 45°C .

- a) $\neg P$ b) $\neg(P \vee Q)$ c) $P \wedge Q$ d) $\neg(\neg P)$ e) $\neg P \wedge \neg Q$ f) $\neg P \vee \neg Q$
- g) $\neg(\neg P \vee \neg Q)$

3) How many rows are needed in the truth table of the given statement formula. $(P \rightarrow Q) \wedge (\neg R \vee S) \Rightarrow T$

Since the given statement formula consisting of P, Q, R, S, T Hence the truth table have 2^5 rows = 32 rows.

4) Negate the statement "for all real numbers x, if $x > 4$ then $x^2 > 16$ ".

For some x, if $x^2 \leq 16$, then $x \leq 4$.