Free and Bound Variables:

1) The variable is said to be bound if it is concerned with either universal $(\forall x)$ or existenti[al] $(\exists x)$ quantifier.

2) The scope of the quantifier is the formulae immediately following the quantifier.

3) The variable which is not concerned with any quantifier is called free variable.

Example: $(\forall x)[P(x,y)]$.

Here x → bound Variable

y → free variable

$P(x,y)$ → Scope of the quantifier.

The Theory of Inference for Predicate Calculus

Rules of Specification — US } Eliminate quantifiers
— ES

Rules of Generalization — UG } prefix the correct quantifier.
— EG

1) Universal Specification: (US - Rule)

$(\forall x)\ P(x) \Rightarrow P(y)$

2) Existential Specification: (ES - Rule)

$(\exists x)\ P(x) \Rightarrow P(y)$

3) Universal Generalization (UG - Rule)

$P(y) \Rightarrow (\forall x)\ P(x)$

4) Existential ~~Spec~~Generalization (EG - Rule)

$P(y) \Rightarrow (\exists x)\ P(x)$

Problems:

1) Show that $(\exists x)\ M(x)$ follows logically from the premises $(\forall x)(H(x) \rightarrow M(x))$ and $(\exists x)(H(x))$

Solution:

Step	Premises	Rule
1	$(\forall x)[H(x) \rightarrow M(x)]$	Rule P
2 {1}	$H(y) \rightarrow M(y)$	Rule US
3	$(\exists x)\ H(x)$	Rule P
4 {3}	$H(y)$	Rule ES
5 {4, 2}	$M(y)$	Rule T ($P, P \rightarrow Q \Rightarrow Q$) Modus Ponens
6 {5}	$(\exists x)\ M(x)$	E(G) Rule EG

2) Verify the Validity of the argument. All humans are mortal. Sachin is a human. Therefore he is mortal.

$H(x)$: x is a human

$M(x)$: x is Mortal

$H(S)$: Sachin is a human.

Soln: The Premises are,

1) $(\forall x)[H(x) \rightarrow M(x)]$ 2) $H(S)$

Conclusion : $M(S)$

Step	Premises	Rule
1.	$(\forall x)[H(x) \rightarrow M(x)]$	Rule P
2 {1}	$H(S) \rightarrow M(S)$	Rule US
3	$H(S)$	Rule P
4 {3, 2}	$M(S)$	Rule T (Modus Ponens).

3) Show that the premises "one student in this class knows how to write programs in JAVA" and "everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job".

Solution: Let $A(x)$: x is in this class

$J(x)$: x knows JAVA program

$H(x)$: x can get high paying jobs.

The premises are

$(\exists x)[A(x) \wedge J(x))$, $(\forall x)[J(x) \rightarrow H(x)]$

Conclusion is $(\exists x)[A(x) \wedge H(x)]$

Step	Premises	Rule
1	$(\exists x)[A(x) \wedge J(x)]$	Rule P
2 {1}	$A(y) \wedge J(y)$	Rule E
3 {2}	$A(y)$	Rule T $(A(y) \wedge J(y) \Rightarrow A(y))$ (Simplification Rule)
4 {2}	$J(y)$	Rule T "
5	$(\forall x)(J(x) \to H(x))$	Rule P
6 {5}	$J(y) \to H(y)$	Rule US
7 {4, 6}	$H(y)$	Rule T (Modus Ponens)
8 {3, 7}	$A(y) \wedge H(y)$	Rule T (Simplification)
9 {8}	$(\exists x)[A(x) \wedge H(x)]$	Rule EG

4) Use CP Rule, if necessary and obtain the following implication

$(\forall x)[P(x) \to Q(x)], (\forall x)[R(x) \to \neg Q(x)] \Rightarrow (\forall x)[R(x) \to \neg P(x)]$

Soln:

Step	Premises	Rule
1	$(\forall x)[P(x) \to Q(x)]$	Rule P
2	$(\forall x)[R(x) \to \neg Q(x)]$	Rule P
3 {2}	$R(y) \to \neg Q(y)$	Rule US
4	$R(y)$	Assumed Premises
5 {3, 4}	$\neg Q(y)$	Rule T (Modus Ponens)
6 {1}	$P(y) \to Q(y)$	Rule US
7 {5, 6}	$\neg P(y)$	Rule T (Modus Tollens)
8 {4, 7}	$R(y) \to \neg P(y)$	Rule CP
9 {8}	$(\forall x)[R(x) \to \neg P(x)]$	Rule UG

5) Verify the validity of the following argument.
"Everything living is a plant or an animal",
"John's gold fish is alive and it is not a plant".
"All animals have hearts". Therefore "John's gold fish has a heart".

$L(x)$: x is a living thing | $L(j)$: j is alive
$P(x)$: x is a plant | $P(j)$: j is not a plant
$A(x)$: x is an animal | $H(j)$: j has a heart.
$H(x)$: x is a heart

Soln: Given:

$(\forall x)[L(x) \to P(x) \vee A(x)]$

$L(j) \wedge \neg P(j)$, $(\forall x)[A(x) \to H(x)]$

Conclusion: $H(j)$

Step	Premises	Rule
1	$(\forall x)[L(x) \to P(x) \vee A(x)]$	Rule P
2 {1}	$L(j) \to P(j) \vee A(j)$	Rule US
3	$L(j) \wedge \neg P(j)$	Rule P
4 {3}	$L(j)$	Rule T $(P \wedge Q \Rightarrow P)$
5 {2,4}	$P(j) \vee A(j)$	Rule T (Modus Ponens)
6 {5}	$\neg P(j) \to A(j)$	Rule T $P \to Q \Leftrightarrow \neg P \vee Q$ MIL
7	$(\forall x)[A(x) \to H(x)]$	Rule P
8 {8}	$A(j) \to H(j)$	Rule US
9	$\neg P(j) \to H(j)$	Rule T
10 {3}	$\neg P(j)$	Rule T. $(P \wedge Q \Rightarrow P, Q)$
11	$H(j)$	Rule T $P, P \to Q \Rightarrow Q$ (Modus Ponens)