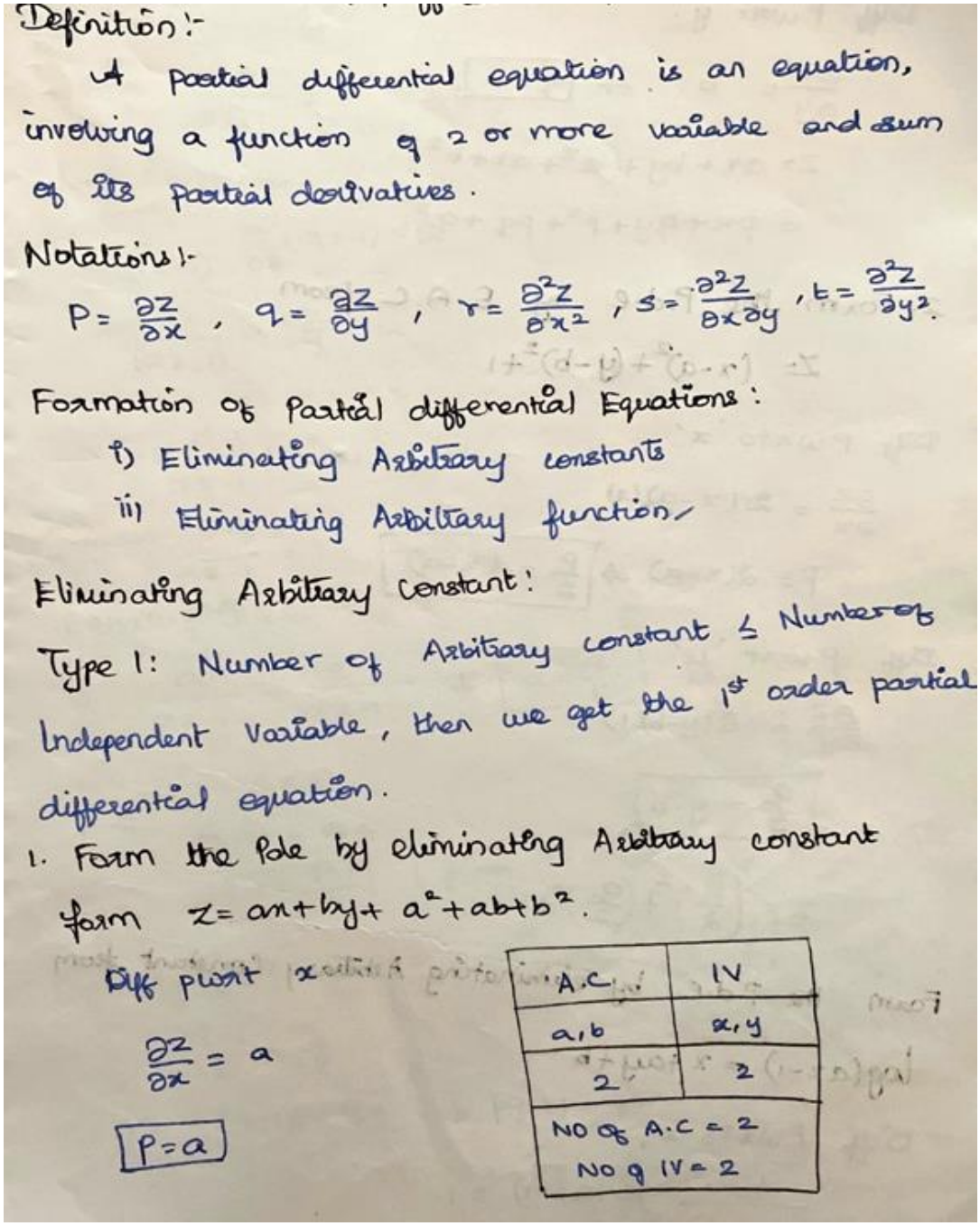

Definition:-

A partial differential equation is an equation, involving a function of 2 or more variable and sum of its partial derivatives.

Notations:-

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}.$$

Formation of Partial differential Equations:

i) Eliminating Arbitrary constants

ii) Eliminating Arbitrary function.

Eliminating Arbitrary constant:

Type 1: Number of Arbitrary constant ≤ Number of Independent Variable, then we get the 1st order partial differential equation.

1. Form the Pde by eliminating Arbitrary constant form $z = ax + by + a^2 + ab + b^2$.

Diff partial x

$$\frac{\partial z}{\partial x} = a$$

$$\boxed{p = a}$$

A.C	IV
a, b	x, y
2	2
NO of A.C = 2	
NO of IV = 2	

Diff p wrt y.

$$\frac{\partial z}{\partial y} = b \Rightarrow \boxed{q = b}.$$

$$z = ax + by + a^2 + ab + b^2.$$

$$= px + qy + p^2 + pq + q^2.$$

2. Form the P.d.e by E.A.C from

$$z = (x-a)^2 + (y-b)^2 + 1$$

Diff p wrt to 'x'.

$$\frac{\partial z}{\partial x} = 2(x-a)(1)$$

$$p = 2(x-a) \Rightarrow \boxed{\frac{p}{2} = (x-a)}$$

Diff p wrt 'y'

$$\frac{\partial z}{\partial y} = 2(y-b)(1)$$

$$\boxed{\frac{q}{2} = y-b}$$

$$z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2 + 1.$$

Form the P.d.e by eliminating Arbitrary Constant from

$$\log(az-1) = x + ay + b$$

Diff p wrt x,

$$\frac{1}{(az-1)} \frac{\partial}{\partial x}(az-1) = 1$$

$$\frac{1}{az-1}\, a\, \frac{\partial z}{\partial x} = 1$$

$$\frac{ap}{az-1} = 1 \longrightarrow (1).$$

Diff P wrt y,

$$\frac{1}{(az-1)}\, \frac{\partial}{\partial y}(az-1) = 0$$

$$\frac{1}{(az-1)}\, a\, \frac{\partial z}{\partial y} = a$$

$$\frac{aq}{az-1} = a$$

$$\frac{qz}{az-1} = 1 \longrightarrow (2).$$

Equating (1) & (2).

$$\frac{ap}{az-1} = \frac{q}{az-1} \Rightarrow \boxed{a = \frac{q}{p}} \longrightarrow (3).$$

$$q = az - 1$$

$$q + 1 = az$$

$$\boxed{a = \frac{q+1}{z}} \quad (\text{from } (2)) \longrightarrow (4)$$

Equating (3) & (4)

$$\frac{q+1}{z} = \frac{q}{p}$$

$$(q+1)p = qz \Rightarrow pq + p = qz$$

$$zq - pq - p = 0.$$

$z = px + qy + \sqrt{p^2+q^2}$

4) $ax + by + \sqrt{a^2+b^2}$

5) $(x+a^2)(y+b^2) \Rightarrow \frac{p^2}{4} + \frac{q^2}{4} + 1$

6) $\frac{(x^2-a^2}{(x-a)^2} + (y-b)^2 + z^2 = 1$

$(p^2+q^2+1)z^2 = 1$