DEFORMATION OF SOLIDS

Strength of Material

When an external force acts on a body, the body tends to undergo some deformation, Due to cohesion between the molecules, the body resists deformation. This resistance by which the material of the body opposes the deformation is known as strength of material.

Elastic limit

Within a certain limit stress $\propto$ strain.
Beyond elastic limit stress $<$ strain.

Intensity of stress (stress)

Within elastic limit stress = applied load, This resisting force per unit area is called stress.

$\sigma = \frac{P}{A}$ unit - N/mm^2 (or) N/m^2

$1\ m = 100\ cm$ $1\ m^2 = 10^4\ cm^2$ $1\ cm^2 = 10^{-4}\ m^2$

$1\ m = 10^3\ mm$ $1\ m^2 = 10^6\ mm^2$ $1\ mm^2 = 10^{-6}\ m^2$

$1\ N/m^2 = 10^{-4}\ N/cm^2 = 10^{-6}\ N/mm^2$

Kilo $= 10^3$

Mega $= 10^1$

Giga $= 10^9$

Terra $= 10^{12}$

1 pascal $= 1\ N/m^2$

Strain

When a body is subjected some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. strain is dimensionless.

Types
1. Tensile strain
2. compressive strain
3. Volumetric strain
4. shear strain.

Tensile strain $= \dfrac{\text{Increase of length}}{\text{Original length}}$

Compressive strain $= \dfrac{\text{Decrease of length}}{\text{Original length}}$

Volumetric strain $= \dfrac{\text{change in volume}}{\text{original volume}}$

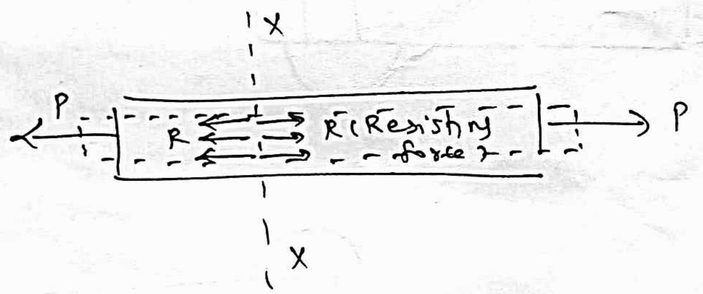
Shear strain → strain produced by shear stress

Types of stresses

Normal stress < Compressive / Tensile

Normal stress is the stress which acts in a direction perpendicular to the area.

Tensile stress

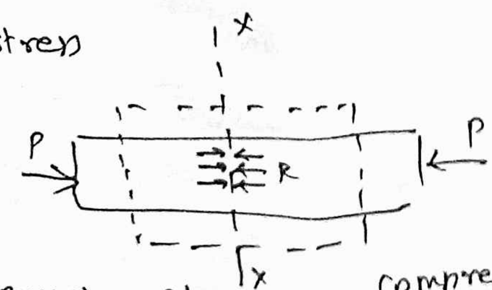


The stress induced in a body, when subjected to two equal and opposite pulls, as a result of which there is an increase in length is known as Tensile stress.

$$\text{Tensile stress} = \sigma = \frac{\text{Tensile load}}{\text{Area}} = \frac{P}{A}$$

$$\text{Tensile strain} \quad e = \frac{dL}{L}$$

Compressive stress



$$\text{Compressive stress} = \frac{\text{compressive load}}{\text{Area}}$$

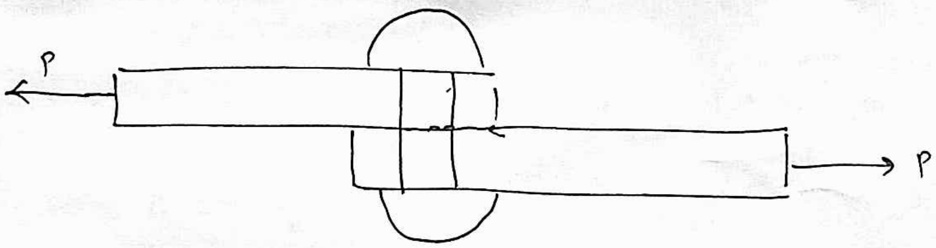
$$\text{Strain} = dL/L$$

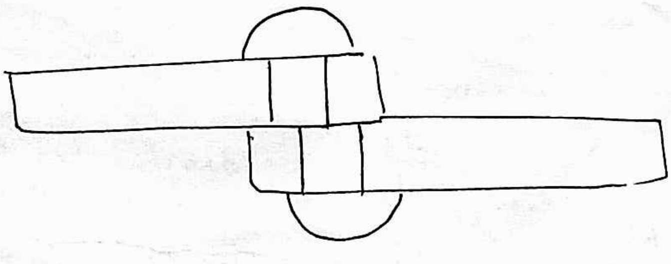
The stress induced in a body when subjected to two equal and opposite pushes, as a result of which there is a decrease in length of the body is known as compressive stress.

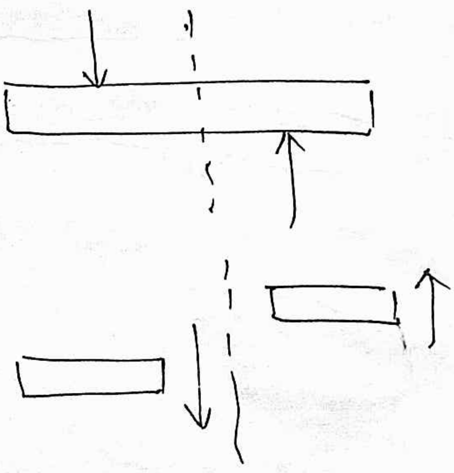
Shear stress (q)

The stress induced in a body when subjected to two equal & opposite forces which are acting tangentially across the resisting section, as a result of which the body tends to shear off across the section is known as 'shear stress'. The corresponding strain is known as 'shear strain'.

shear stress is the stress which acts tangential to the area







$$\text{Shear stress } q = \frac{\text{shear resistance}}{\text{shear Area}}$$

Simple stress

A single stress which acts on a section is known as simple stress. It may be Tensile / compressive / shear stress.

Compound stress

If more than one stress is acting on a section, then it is called as compound stress.

Principal planes

The planes, which carry no shear stress are known as principal planes. These planes carry only normal stresses.

Principal stresses

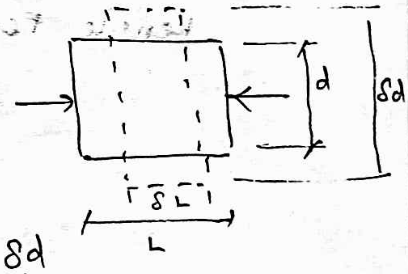
The normal stresses acting on a principal plane are known as principal stresses.

Relationship between stress & strain (1D)

$$E = \frac{\sigma}{e}$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

2D



$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ (or) } \frac{\delta d}{d}$$

$$\text{Poisson ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu = \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

1. A concrete cylinder of diameter 150 mm and length 300 mm when subjected to an axial compressive load of 240 kN resulted in an increase of diameter by 0.127 mm and a decrease in length of 0.28 mm. Compute the value of Poisson's ratio and modulus of elasticity E.

Ans. $\frac{1}{m} = 0.907$

$E = 14.05 \ GN/mm^2$

2. For a given material $E = 110 \ GN/m^2$ and shear modulus is $42 \ GN/m^2$. Find the bulk modulus and lateral contraction of round bar of 37.5 mm diameter and 2.4 m length when stretched 2.5 mm.

Ans $\delta d = 0.0121$ mm

3. The following data relate to a bar subjected to a tensile test.

Diameter of the bar = 30 mm

Tensile load = 54 kN

Gauge length = 300 mm

Extension of the bar = 0.112 mm

Change in dia = 0.00366 mm

Calculate 1) Poisson's ratio

2) Three modulus values

Ans $\frac{1}{m} = 0.327$

$E = 2.05 \times 10^5 \ MN/m^2$

$C = 0.77 \times 10^5 \ MN/m^2$

$K = 1.97 \times 10^5 \ MN/m^2$

Hooke's law

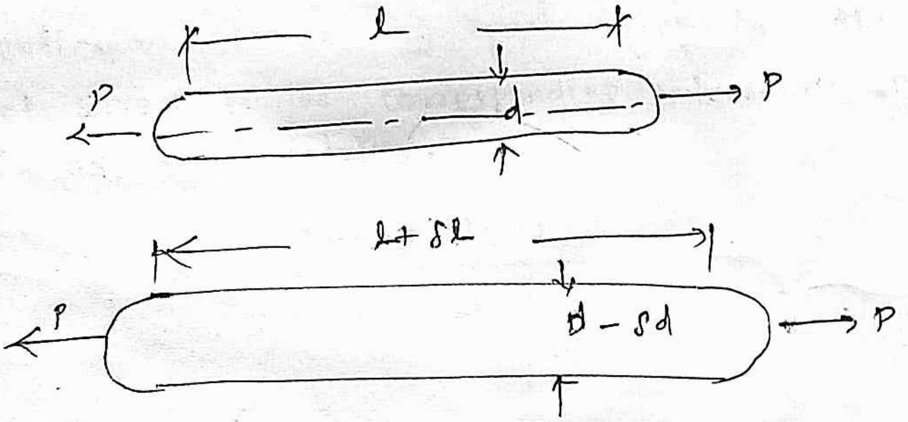
It states "when a material is loaded, within its elastic limit, the stress is proportional to the strain".

Elastic constants

strains in a body, when subjected to a Direct stress

1. Primary | Linear strain
2. secondary / Lateral strain

Linear strain & Lateral strain



$$\text{Linear strain} = \frac{\delta l}{l}$$

$$\text{Linear strain} = \frac{\text{change in linear dimension}}{\text{orginal dimension.}}$$

ie Deformation of the bar per unit length in the direction of force.

Lateral strain

Deformation of the bar in the direction right angles to the direction of force

$$\text{Lateral strain} = \frac{\text{change in lateral dimensions}}{\text{orginal dimension}}$$

$$= \frac{\delta b}{b} \text{ (or) } \frac{\delta d}{d}$$

Poisson Ratio $(\mu = \frac{1}{m})$

$$\mu = \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

Value of Poisson's ratio

steel – 0.25 to 0.33

C.I – 0.23 to 0.27

Copper – 0.31 to 0.34

Brass – 0.32 to 0.42

Aluminium – 0.32 to 0.36

Bulk modulus (K)

When a body is subjected to three mutually perpendicular stresses of equal intensity, the ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$= \frac{\sigma}{dv/v}$$

Relationship between Young's modulus & Bulk modulus

$$E = 3K\left(1 - \frac{2}{m}\right)$$

Modulus of rigidity (C cos G)

$$C = \frac{\text{shear stress}}{\text{shear strain}}$$

Relationship between Young's modulus & Modulus of rigidity

$$C = \frac{mE}{2(m+1)}$$