Type 1

It is of the form $f(p,q) = 0$.

Complete Integral

Here the solution is $z = ax + by + c$

(1) Solve: $p^2 + q^2 = 4$

Here solution is $z = ax + by + c$.

$f(p,q) = p^2 + q^2 - 4 = 0$

$\Rightarrow f(a,b) = a^2 + b^2 - 4 = 0$

$\Rightarrow a^2 + b^2 = 4$

$\Rightarrow b^2 = 4 - a^2$

$b = \pm\sqrt{4-a^2}$.

$\therefore$ Complete soln. is $z = ax \pm (\sqrt{4-a^2})y + c$.

(2) Solve: $\sqrt{p} + \sqrt{q} = 1$

Soln. is $z = ax + by + c$.

$f(p,q) = \sqrt{p} + \sqrt{q} - 1 = 0$

$f(a,b) = \sqrt{a} + \sqrt{b} - 1 = 0$

$\Rightarrow \sqrt{a} + \sqrt{b} = 1$

$\Rightarrow b = (1-\sqrt{a})^2$

$z = ax + (1-\sqrt{a})^2 y + c$.

(3) $p+q+pq=0$

$f(p,q) = p+q+pq = 0$

$f(a,b) = a+b+ab = 0$

$\Rightarrow a+b+ab=0$

$\Rightarrow a(1+b)+b=0$

$\Rightarrow b = -a(1+b)$

$\Rightarrow a = -\frac{b}{1+b}$

$\therefore z = -\frac{b}{(1+b)}x + by + c$

(5) $p = q^2$

$f(p,q) = p-q^2 = 0$

$f(a,b) = a-b^2 = 0$

$\Rightarrow a = b^2.$

$\therefore z = b^2x + by + c.$