SNS COLLEGE OF TECHNOLOGY

23MCB204 – SOLID MECHANICS

UNIT I - SIMPLE STRESSES AND STRAINS

1.1. Definition

In day-to-day work, an engineer comes across certain materials, *i.e.*, steel girders, angle irons, circular bars, cement etc., which are used in his projects. While selecting a suitable material, for his project, an engineer is always interested to know its strength. The strength of a material may be defined as ability, to resist its failure and behaviour, under the action of external forces. It has been observed that, under the action of these forces, the material is first deformed and then its failure takes place. A detailed study of forces and their effects, alongwith some suitable protective measures for the safe working conditions, is known as Strength of Materials. As a matter of fact, such

1.2. Fundamental Units

The measurements of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units. All the physical quantities, met with in Strength of Materials, are expressed in terms of the following three fundamental quantities :

1. Length, 2. Mass and 3. Time.

1.3. Derived Units

Sometimes, physical quantities are expressed in other units, which are derived from fundamental units, known as derived units, *e.g.*, units of area, velocity, acceleration, pressure, etc.

1.4. Systems of Units

Following are only four systems of units, which are commonly used and universally recognised.

1. C.G.S. units, 2. F.P.S. units, 3. M.K.S. units and 4. S.I. units.

1.5. S.I. Units (International System of Units)

The eleventh General Conference* of Weights and Measures has recommended a unified and systematically constituted system of fundamental and derived units for international use. This system of units is now being used in many countries. In India, the Standards of Weights and Measures Act of 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, the †fundamental units are metre (m), kilogram (kg) and second (s) respectively. But there is a slight variation in their derived units. The following derived units will be used in this book :

Density (or Mass density)	kg/m^3
Force (in Newtons)	N (= $kg.m/s^2$)
Pressure (in Pascals)	Pa (= $N/m^2 = 10^{-6}\ N/mm^2$)
Stress (in Pascals)	Pa (=$N/m^2 = 10^{-6}\ N/mm^2$)
Work done (in Joules)	J (= N-m)
Power (in Watts)	W (= J/s)

TABLE 1.1.

Factor by which the unit is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1 000 000 000 000	10^{12}	Tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

1.12. Algebra

1. $a^0 = 1$; $x^0 = 1$

   (*i.e., Anything raised to the power zero is one.*)

2. $x^m \times x^n = x^{m+n}$

   (*i.e., If the bases are same, in multiplication, the powers are added.*)

3. $\dfrac{x^m}{x_n} = x^{m-n}$

   (*i.e., If the bases are same, in division, the powers are subtracted.*)

4. If $ax^2 + bx + c = 0$

   then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

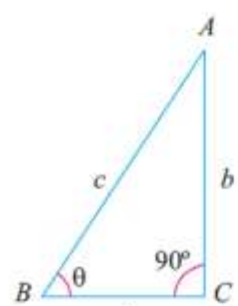
   where $\quad a$ is the coefficient of x^2,

   b is the coefficient of x and c is the constant term.

1.13. Trigonometry

In a right-angled triangle ABC as shown in Fig. 1.1.

1. $\dfrac{b}{c} = \sin\theta$
2. $\dfrac{c}{a} = \cos\theta$
3. $\dfrac{b}{a} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$
4. $\dfrac{c}{b} = \dfrac{1}{\sin\theta} = \text{cosec}\,\theta$
5. $\dfrac{c}{a} = \dfrac{1}{\cos\theta} = \sec\theta$



1.16. Scalar Quantities

The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

1.17. Vector Quantities

The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc. Following are the important features of vector quantities :

1. **Representation of a vector.** A vector is represented by a directed line as shown in Fig. 1.2. It may be noted that the length *OA* represents the magnitude of the vector $\overrightarrow{OA}$. The direction of the vector is $\overrightarrow{OA}$ is from *O* (*i.e.*, starting point) to *A* (*i.e.*, end point). It is also known as vector *P*.

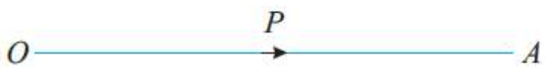


Fig. 1.2. Vector $\overrightarrow{OA}$

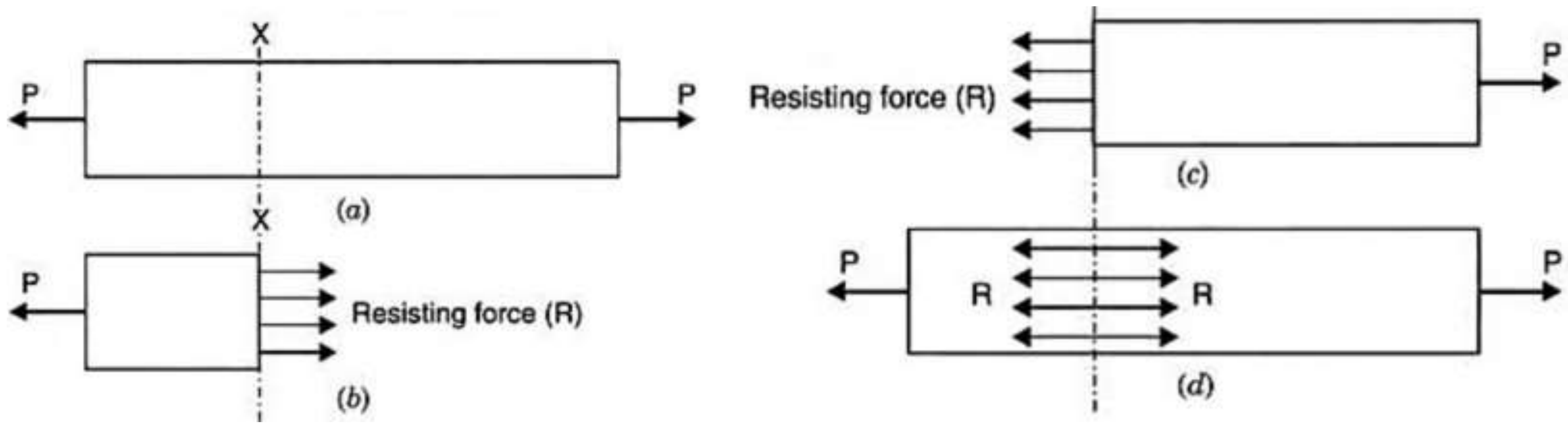
2. **Unit vector.** A vector, whose magnitude is unity, is known as unit vector.
3. **Equal vectors.** The vectors, which are parallel to each other and have same direction (*i.e.*, same sense) and equal magnitude are known as equal vectors.
4. **Like vectors.** The vectors, whch are parallel to each other and have same sense but unequal magnitude, are known as like vectors.

Types of Stresses:

The stress may be normal stress or a shear stress. Normal stress is the stress which acts in a direction perpendicular to the area. It is represented by ζ (sigma). The normal stress is further divided into tensile stress and compressive stress.

Tensile Stress:

The stress induced in a body, when subjected to two equal and opposite pulls as shown in Figure as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is known as *tensile strain.* The tensile stress acts normal to the area and it pulls on the area.



Tensile Stress:

$$\therefore \text{Tensile stress} = \sigma = \frac{\text{Resisting force }(R)}{\text{Cross-sectional area}} = \frac{\text{Tensile load }(P)}{A}$$

$$\sigma = \frac{P}{A}$$

And tensile strain is given by,

$$e = \frac{\text{Increase in length}}{\text{Original length}} = \frac{dL}{L}.$$

Units of Stress

The unit of stress depends upon the unit of load (or force) and unit of area. In M.K.S. units, the force is expressed in kgf and area in meter square (i.e., m^2). Hence unit of stress becomes as kgf/m^2. If area is expressed in centimeter square *(i.e., cm^2), the stress is expressed as kgf/cm^2*.

In the S.I. units, the force is expressed in newtons (written as N) and area is expressed as m^2. Hence unit of stress becomes as N/m^2. The area is also expressed in millimeter square then unit of force becomes as N/mm^2

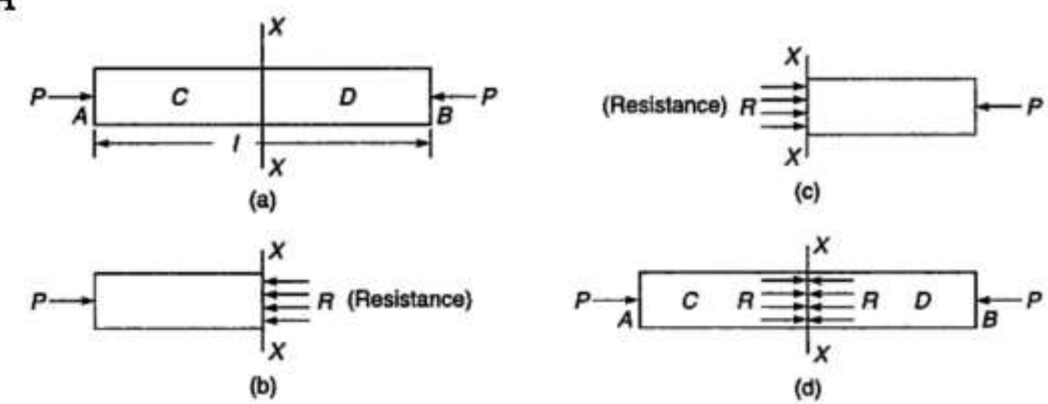
$1\ N/m^2 = 1\ N/(100cm)^2 = 1\ N/(10^4\ cm^2) = 10^{-4}\ N/cm^2$ or $10^{-6}\ N/mm^2$ Or $1\ MPa = 1\ N/mm^2$

Compressive Stress:

The stress induced in a body, when subjected to two equal and opposite pushes as shown in Figure as a result of which there is a decrease in length of the body, is known as compressive stress. And the ratio of decrease in length to the original length is known as compressive strain. The compressive stress acts normal to the area and it pushes on the area.

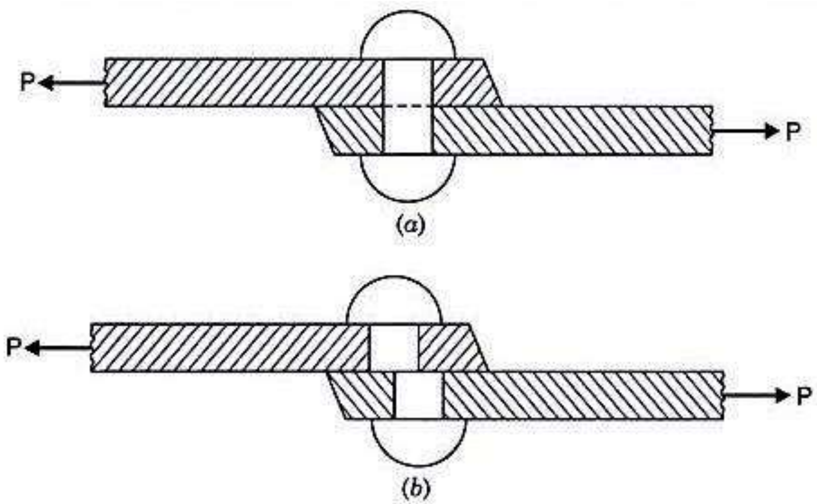
$$\sigma = \frac{\text{Resisting Force } (R)}{\text{Area } (A)} = \frac{\text{Push } (P)}{\text{Area } (A)} = \frac{P}{A}$$

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L}$$



Shear Stress:

The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in Figure as a result of which the body tends to shear off across the section, is known as shear stress. The corresponding strain is known as shear strain. The shear stress is the stress which acts tangential to the area. It is represented by τ.



STRAIN

When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Strain is dimensionless. Strain may be :

- ❖Tensile strain / Compressive strain,
- ❖Volumetric strain
- ❖Shear strain.

- If there is some increase in length of a body due to external force, then the ratio of increase of length to the original length of the body is known as tensile strain.
- But if there is some decrease in length of the body, then the ratio of decrease of the length of the body to the original length is known as compressive strain.
- The ratio of change of volume of the body to the original volume is known as volumetric strain.
- The strain produced by shear stress is known as shear strain.

LINEAR ELASTICITY AND ELASTIC LIMIT

- When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its original shape and size (which means the deformation disappears completely), the body is known as elastic body.
- This property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity. The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus there is a limiting value of force up to and within which, the deformation completely disappears on the removal of the force.
- The value of stress corresponding to this limiting force is known as the elastic limit of the material.
- If the external force is so large that the stress exceeds the elastic limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in the material.

HOOKE'S LAW

For elastic bodies, the ratio of stress to strain is constant and is known as Young's modulus or the modulus of elasticity and is denoted by E, i.e.,

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \quad \text{or} \quad \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

$$E = \frac{\sigma}{e}$$

Strain has no units as it is a ratio. Thus, E has the same units as stress.

- Similarly, for elastic materials, the shear strain is found to be proportional to the applied shear stress within the elastic limit. Modulus of rigidity or shear modulus denoted by G is the ratio of shear stress to shear strain, i.e.,

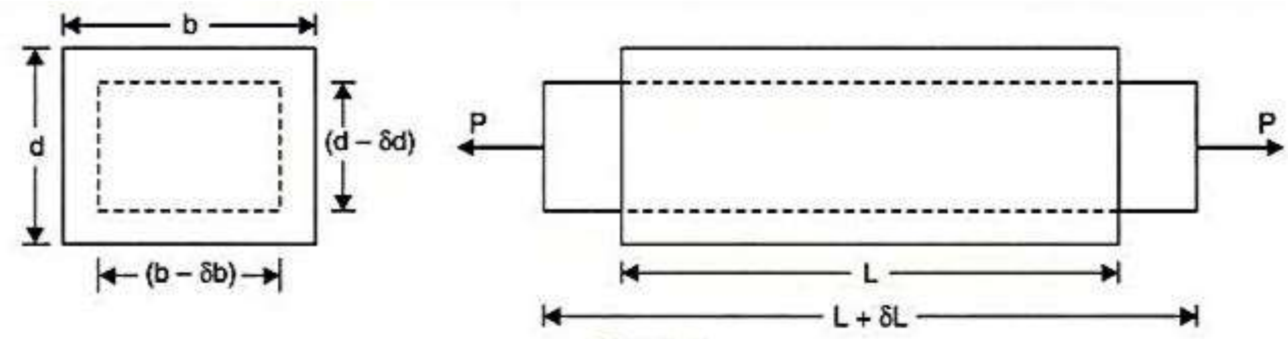
- The ratio between the volumetric (Identical) stress and the volumetric strain is called Bulk modulus of elasticity and is denoted by K.

POISSON’S RATIO

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ or ν or 1/m. Hence mathematically,

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Longitudinal strain & Lateral strain:



$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$