SNS COLLEGE OF TECHNOLOGY

23MCB204 – SOLID MECHANICS

UNIT I - SIMPLE STRESSES AND STRAINS

3.2. Types of Bars of Varying Sections

Though there are many types of bars of varying sections, in the field of strength of materials yet the following are important from the subject point of view :

1. Bars of different sections
2. Bars of uniformly tapering sections
3. Bars of composite sections.

3.3. Stresses in the Bars of Different Sections

Sometimes a bar is made up of different lengths having different cross-sectional areas as shown in Fig. 3.1.

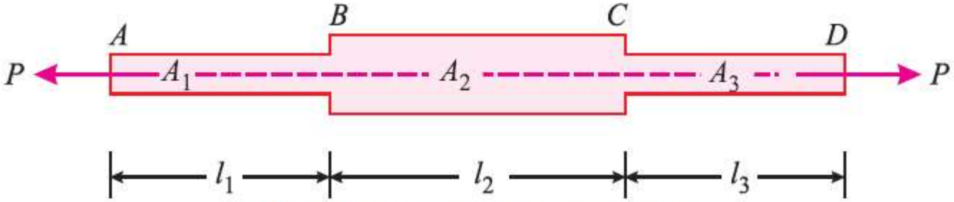

Fig. 3.1. Bars of different sections

In such cases, the stresses, strains and hence changes in lengths for each section is worked out separately as usual. The total changes in length is equal to the sum of the changes of all the individual lengths. It may be noted that each section is subjected to the same external axial pull or push.

Let P = Force acting on the body,

E = Modulus of elasticity for the body,

l_1 = Length of section 1,

A_1 = Cross-sectional area of section 1,

l_2, A_2 = Corresponding values for section 2 and so on.

We know that the change in length of section 1.

$$\delta l_1 = \frac{Pl_1}{A_1E} \quad \text{Similarly} \quad \delta l_2 = \frac{Pl_2}{A_2E} \quad \text{and so on}$$

∴ Total deformation of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 + \ldots\ldots\ldots$$

$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E} + \ldots\ldots\ldots$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \ldots\ldots\ldots\right)$$

EXAMPLE 3.4. *A compound bar ABC 1.5 m long is made up of two parts of aluminium and steel and that cross-sectional area of aluminium bar is twice that of the steel bar. The rod is subjected to an axial tensile load of 200 kN. If the elongations of aluminium and steel parts are equal, find the lengths of the two parts of the compound bar. Take E for steel as 200 GPa and E for aluminium as one-third of E for steel.*

SOLUTION. Given: Total length $(L) = 1.5$ m $= 1.5 \times 10^3$ mm ; Cross-sectional area of aluminium bar $(A_A) = 2\,A_S$; Axial tensile load $(P) = 200$ kN $= 200 \times 10^3$ N ; Modulus of elasticity of steel $(E_S) = 200$ GPa $= 200 \times 10^3$ N/mm^2 and modulus of elasticity of aluminium $(E_A) = \dfrac{E_S}{3} = \dfrac{200 \times 10^3}{3}$ N/mm^2.

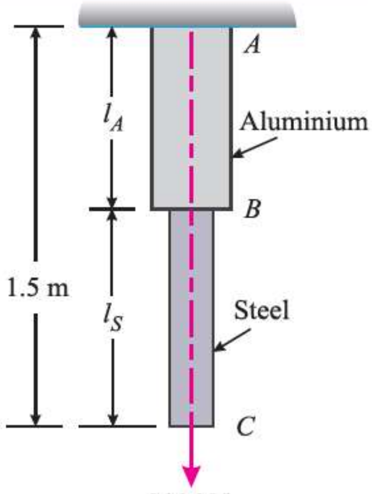

Fig. 3.5

Let, l_A = Length of the aluminium part,

and l_S = Length of the steel part.

We know that elongation of the aluminium part *AB*,

$$\delta l_A = \frac{P.l_A}{A_A.E_A} = \frac{(200 \times 10^3) \times l_A}{2A_S \times \left(\dfrac{200 \times 10^3}{3}\right)}$$

$$= \frac{1.5\,l_A}{A_S} \qquad ...(i)$$

and elongation of the steel part *BC*,

$$\delta l_S = \frac{P.l_S}{A_S.E_S} = \frac{(200 \times 10^3) \times l_S}{A_S \times (200 \times 10^3)} = \frac{l_S}{A_S} \qquad ...(ii)$$

Since elongations of aluminium and steel parts are equal, therefore equating equations (*i*) and (*ii*),

$$\frac{1.5\,l_A}{A_S} = \frac{l_S}{A_S} \quad \text{or} \quad l_S = 1.5\,l_A$$

We also know that total length of the bar *ABC* (*L*)

$$1.5 \times 10^3 = l_A + l_S = l_A + 1.5\,l_A = 2.5\,l_A$$

$$l_A = \frac{1.5 \times 10^3}{2.5} = 600 \text{ mm} \qquad \textbf{Ans.}$$

$$l_S = (1.5 \times 10^3) - 600 = 900 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 3.5. *An alloy circular bar ABCD 3 m long is subjected to a tensile force of 50 kN as shown in Fig. 3.6.*

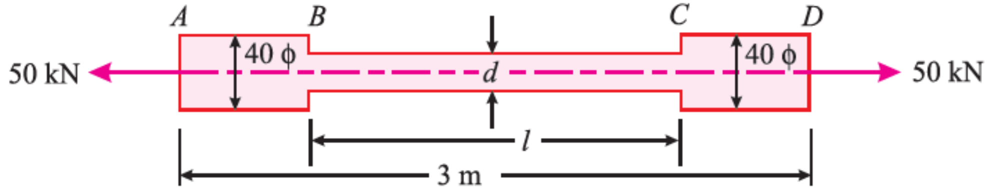

Fig. 3.6

If the stress in the middle portion BC is not to exceed 150 MPa, then what should be its diameter? Also find the length of the middle portion, if the total extension of the bar should not exceed by 3 mm. Take E as 100 GPa.

SOLUTION. Total length of circular bar (L) = 3m = 3×10^3 mm = 3000 mm ; Tensile force (P) = 50 kN = 50×10^3 N ; Maximum stress of portion BC (σ_{BC}) = 150 MPa = 150 N/mm^2 ; Total extension (δl) = 3 mm and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm^2.

Diameter of the middle portion BC

Let d = Diameter of the middle portion in mm.

We know that stress in the middle portion BC (σ_{BC}),

$$150 = \frac{P}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} \times (d)^2} = \frac{63.66 \times 10^3}{d^2}$$

$$\therefore \quad d^2 = \frac{63.66 \times 10^3}{150} = 424.4 \quad \text{or} \quad d = 20.6 \text{ mm} \quad \textbf{Ans.}$$

length of the middle portion BC

Let l_{BC} = Length of the middle portion in mm.

We know that area of the end portions AB and CD, We know that area of the end portions *AB* and *CD*,

$$A_1 = \frac{\pi}{4} \times (40)^2 = 1257 \text{ mm}^2$$

and area of the middle portion *BC*,

$$A_2 = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (20.6)^2 = 333.3 \text{ mm}^2$$

We also know that total extension of bar (δl),

$$3 = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right] = \frac{50 \times 10^3}{100 \times 10^3} \times \left[\frac{3000 - l}{1257} + \frac{l}{333.3}\right]$$

$$= 0.5\,[2.387 - 0.0008\,l + 0.003\,l] = 0.5\,[2.387 + 0.0022\,l]$$

$$= 1.194 + 0.0011\,l$$

$$\therefore \quad l = \frac{3 - 1.194}{0.0011} = 1.64 \times 10^3 \text{ mm} = 1.64 \text{ m}$$ **Ans.**

3.7. Stresses in the Bars of Composite Structures

A bar made up of two or more different materials, joined together is called a composite bar. The bars are joined in such a manner, that the system extends or contracts as one unit, equally, when subjected to tension or compression. Following two points should always be kept in view, while solving example on composite bars :

1. Extension or contraction of the bar is equal. Therefore strain (*i.e.*, deformation per unit length) is also equal.
2. The total external load, on the bar, is equal to the sum of the loads carried by the different materials.

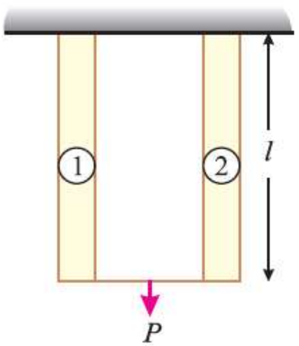

Fig. 3.21

Consider a composite bar made up of two different materials as shown in Fig. 3.21.

Let P = Total load on the bar,

l = Length of the bar 1

l_2 = Length of the bar 2

A_1 = Area of bar 1,

E_1 = Modulus of elasticity of bar 1.

P_1 = Load shared by bar 1, and

A_2, E_2, P_2 = Corresponding values for bar 2,

We know that total load on the bar,

$$P = P_1 + P_2 \quad ...(i)$$

∴ Stress in bar 1, $$\sigma_1 = \frac{P_1}{A_1}$$

and strain in bar 1, $$\varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{P_1}{A_1 E_1}$$

∴ Elongation, $$\delta l_1 = \varepsilon_1 . l_1 = \frac{\sigma_1 l_1}{E_1} = \frac{P_1 l_1}{A_1 E_1} \quad ...(ii)$$

Similarly, elongation of bar 2,

$$\delta l_2 = \varepsilon_2 l_2 = \frac{\sigma_2 l_2}{E_1} = \frac{P_2 l_2}{A_2 E_2} \quad ...(iii)$$

Since both the elongations are equal, therefore equating (*ii*) and (*iii*), we get $\delta l_1 = \delta l_2$

$$\frac{P_1 l}{A_1 E_1} = \frac{P_2 l}{A_2 E_2} \quad \text{or} \quad \frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2} \quad ...(iv)$$

or
$$P_2 = P_1 \times \frac{A_2 E_2}{A_1 E_1}$$

But
$$P = P_1 + P_2 = P_1 + P_1 \times \frac{A_2 E_2}{A_1 E_1}$$

$$= P_1 \left(1 + \frac{A_2 E_2}{A_1 E_1}\right) = P_1 \left(\frac{A_1 E_1 + A_2 E_2}{A_1 E_1}\right)$$

or
$$P_1 = P \times \frac{A_1 E_1}{A_1 E_1 + A_2 E_2} \quad ...(v)$$

Similarly,
$$P_2 = P \times \frac{A_2 E_2}{A_1 E_1 + A_2 E_2} \quad ...(vi)$$

From these equations we can find out the loads shared by the different materials. We have also seen in equation (*iv*) that

$$\frac{Pl_1}{A_1 E_1} = \frac{Pl_2}{A_2 E_2}$$

or $$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \qquad \cdots \left(\because \frac{P}{A} = \sigma = \text{Stress}\right)$$

$\therefore$ $$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2 \qquad \ldots(vii)$$

Similarly, $$\sigma_2 = \frac{E_2}{E_1} \times \sigma_1 \qquad \ldots(viii)$$

From the above equations, we can find out the stresses in the different materials. We also know that the total load,

$$P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$$

From the above equation, we can also find out the stress in the different materials.

EXAMPLE 3.16. *A reinforced concrete column 500 mm × 500 mm in section is reinforced with 4 steel bars of 25 mm diameter, one in each corner. The column is carrying a load of 1000 kN. Find the stresses in the concrete and steel bars. Take E for steel = 210 GPa and E for concrete = 14 GPa.*

SOLUTION. Given : Area of column = 500 × 500 = 2,50,000 mm^2; No. of steel bars (n) = 4 ; Diameter of steel bars (d) = 25 mm ; Load on column (P) = 1,000 kN = 1,000 × 10^3 N ; Modulus of elasticity of steel (E_S) = 210 GPa and modulus of elasticity of concrete (E_C) = 14 GPa.

Let σ_S = Stress in steel, and

σ_C = Stress in concrete.

We know that area of steel bars,

$$A_S = 4 \times \frac{\pi}{4} \times (d)^2 \text{ mm}^2 \quad \ldots(i)$$

$$= 4 \times \frac{\pi}{4} \times (25)^2 = 1963 \text{ mm}^2$$

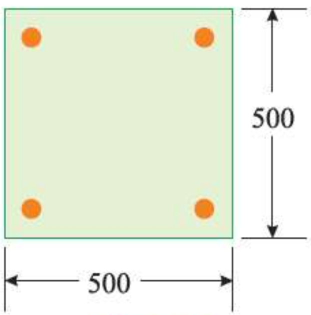

Fig. 3.23

∴ Area of concrete, $A_C = 250,000 - 1963 \text{ mm}^2$

$= 248\,037 \text{ mm}^2$

We also know that stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = \frac{210}{14} \times \sigma_C = 15\,\sigma_C \quad \ldots(ii)$$

and total load (P), $1,000 \times 10^3 = (\sigma_S \,.\, A_S) + (\sigma_C \,.\, A_C)$

$= (15\,\sigma_C \times 1963) + (\sigma_C \times 248\,037) = 277\,482\,\sigma_C$

$$\sigma_C = \frac{1,000 \times 10^3}{277\,482} = 3.6 \text{ N/mm}^2 = 3.6 \text{ MPa} \quad \textbf{Ans.}$$

and $\sigma_S = 15\,\sigma_C = 15 \times 3.6 = 54$ MPa **Ans.**