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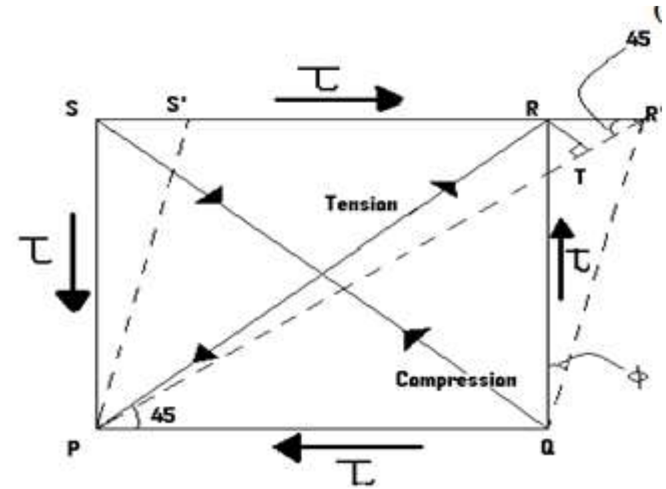
23MCB204 – SOLID MECHANICS

UNIT I - SIMPLE STRESSES AND STRAINS

Consider a solid cube, subjected to a Shear Stress on the faces PQ and RS and complimentary Shear Stress on faces QR and PS. The distortion of the cube, is represented by the dotted lines. The diagonal PR distorts to PR'.



(a) Relationship between E and G

Modulus of Rigidity, G = $\frac{Shear Stress}{Shear strain}$

Shear Strain = $\frac{Shear stress}{G}$

From the diagram, Shear Strain ϕ = $\frac{PR'}{QR}$

Since Shear Stress = τ ,

$$\frac{RR'}{QR} = \frac{\tau}{G} \ldots\ldots (i)$$

From R, drop a perpendicular onto distorted diagonal PR'

The strain experienced by the diagonal = $\frac{TR'}{PR} (Considering\ that\ PT \approx PR)$

= $\frac{RR' cos45}{(QR/cos45)} = \frac{RR'}{2QR}$

Strain of the Diagonal PR = $\frac{RR'}{2QR} = \frac{\tau}{2G} (From I) \ldots\ldots\ldots (ii)$

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Let f be the Direct Stress induced in the diagonal PR due to the Shear Stress τ

Strain of the diagonal = $\frac{\tau}{2G} = \frac{f}{2G} \ldots\ldots\ldots\ldots (iii)$

The diagonal PR is subjected to Direct Tensile Stress while the diagonal RS is subjected to Direct Compressive Stress.

The total strain on Diagonal PR would be = $\frac{f}{E} + \frac{1}{m}(\frac{f}{E})$

$= \frac{f}{E}(1 + \frac{1}{m}) \ldots\ldots\ldots\ldots (iv)$

Comparing Equations (III) and (IV), we have

$\frac{f}{2G} = \frac{f}{E}(1 + \frac{1}{m})$

Re – arranging the terms, we have,

$$E = 2G(1 + \frac{1}{m}) \ldots\ldots\ldots\ldots (A)$$

(b) Relationship between E and K

Instead of Shear Stress , let the cube be subjected to direct stress f on all faces of the cube.

We know,

$$e_v = \frac{f_x+f_y+f_z}{E}[1 - \frac{2}{m}]$$

Since $f = f_x = f_y = f_z$

$$e_v = \frac{3f}{E}[1 - \frac{2}{m}] \ldots\ldots\ldots\ldots (v)$$

Also, by the definition of Bulk Modulus,

$$e_v = \frac{f}{K} \ldots\ldots\ldots\ldots (vi)$$

Equating (V) and (VI), we have:

$$\frac{f}{K} = \frac{3f}{E}[1 - \frac{2}{m}]$$

$$E = 3K[1 - \frac{2}{m}] \ldots\ldots\ldots\ldots (B)$$

(c) Relationship between E, G and K

From the equation **(A)**,

$$\frac{1}{m} = \frac{E-2G}{2G}$$

From the equation **(B)**

$$\frac{1}{m} = \frac{3K-E}{6K}$$

Equating both, we get,

$$\frac{E-2G}{2G} = \frac{3K-E}{6K}$$

Simplifying the equation, we get,

$$E = \frac{9KG}{3K+G}$$

This is the relationship between E, G and K.