Even and odd signal :-

Even signal :- (Symmetric signal)

A signal is said to be an even signal if inversion of time axis doesnot change the amplitude

CT : $x(t) = x(-t)$

DT : $x(n) = x(-n)$

(eg) cosine wave $\cos(-\theta) = \cos\theta$

ODD signal :- [Anti symmetric signals]

A signal is said to be an odd signal if inversion of time axis also inverse the amplitude of the signal

CT : $x(-t) = -x(t)$

DT : $x(-n) = -x(n)$

(Eg) Sine wave $\sin(-\theta) = -\sin\theta$

Representation of signals in Even and odd parts :-

$$x(t) = x_e(t) + x_o(t) \rightarrow (1)$$

$$x(-t) = x_e(t) - x_o(t) \rightarrow (2)$$

Even : (1) + (2)

$$x(t) + x(-t) = 2x_e(t)$$

$$\therefore x_e(t) = \frac{x(t) + x(-t)}{2}$$

ODD : (1) - (2)

$$x(t) - x(-t) = 2x_o(t)$$

$$\therefore x_o(t) = \frac{x(t) - x(-t)}{2}$$

Discrete :-

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

Problems :-

1) find the even and odd components of the signal :-

$$x(t) = \cos t + \sin t$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x(-t) = \cos(-t) + \sin(-t)$$

$$x(-t) = \cos t - \sin t$$

$$x_e(t) = \frac{\cos t + \sin t + \cos t - \sin t}{2}$$

$$x_e(t) = \frac{2\cos t}{2}$$

$$\boxed{x_e(t) = \cos t}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\cos t + \sin t - (\cos t - \sin t)}{2}$$

$$= \frac{\cos t + \sin t - \cos t + \sin t}{2}$$

$$= \frac{2\sin t}{2}$$

$$\boxed{x_o(t) = \sin t}$$

2) $x(t) = \cos t + \sin t + \cos t \cdot \sin t$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$$

$$= \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \frac{\cos t + \sin t + \cos t \sin t + \cos t - \sin t - \cos t \sin t}{2}$$

$$x_e(t) = \frac{2\cos t}{2}$$

$$\boxed{x_e(t) = \cos t}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\cos t + \sin t + \cos t \sin t - [\cos t - \sin t - \cos t \sin t]}{2}$$

$$= \frac{\cancel{\cos t} + \sin t + \cos t \sin t - \cancel{\cos t} + \sin t + \cos t \sin t}{2}$$

$$= \frac{\cancel{2}[\sin t + \cos t \sin t]}{\cancel{2}} = \sin t [1 + \cos t]$$

(3) $x(t) = \sin t + 2\sin t + 2\sin^2 t \cos t$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$x(t) = \cancel{\sin t} + 2\cancel{\sin t} + \cancel{2}\left(\frac{1-\cos 2t}{\cancel{2}}\right)\cos t$

$x(-t) = \sin(-t) + 2\sin(-t) + \cos(t) - \cos(t)\cos 2t$

$x(-t) = -\cancel{\sin t} - 2\cancel{\sin t} + \cos t - \cos t \cos 2t$

$$x_e(t) = \frac{2\cos t - 2\cos t \cos 2t}{2} \Rightarrow \frac{\cancel{2}[\cos t - \cos t \cos 2t]}{\cancel{2}}$$

$$\boxed{x_e(t) = \cos t [1 + \cos 2t]}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$x(t) = \sin t + 2\sin t + \cancel{\cos t} - \cos 2t \cos t$

$-x(-t) = \sin t + 2\sin t - \cancel{\cos t} + \cos 2t \cos t$

$$x_o(t) = \frac{\cancel{2}[\sin t + 2\sin t]}{\cancel{2}}$$

$$\boxed{x_o(t) = 3\sin t}$$

④ Find the Even and odd components of the signal :-

$x(n) = \{3, 2, \underset{\uparrow}{1}, 4, 5\}$

index value

$x(-2) = 3,\ x(-1) = 2,\ x(0) = 1,\ x(1) = 4,\ x(2) = 5$

Even :-

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$n = -2$ $\quad x_e(-2) = \frac{x(-2) + x(2)}{2} = \frac{3+5}{2} = 8/2 = 4$

$n = -1$ $\quad x_e(-1) = \frac{x(-1) + x(1)}{2} = 6/2 = 3$

$n = 0$ $\quad x_e(0) = \frac{x(0) + x(0)}{2} = 2/2 = 1$

$n = 1$ $\quad x_e(1) = \frac{x(1) + x(-1)}{2} = 6/2 = 3$

$n = 2$ $\quad x_e(2) = \frac{x(2) + x(-2)}{2} = 8/2 = 4$

$x_e(n) = \{4, 3, \underset{\uparrow}{1}, 3, 4\}$

odd :-

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$n = -2$ $\quad x_o(-2) = \frac{x(-2) - x(2)}{2} = \frac{3-5}{2} = -2/2 = -1$

$n = -1$ $\quad x_o(-1) = \frac{x(-1) - x(1)}{2} = -2/2 = -1$

$n = 0$ $\quad x_o(0) = \frac{x(0) - x(0)}{2} = 1-1/2 = 0$

$n = 1$ $\quad x_o(1) = \frac{x(1) - x(-1)}{2} = 2/2 = 1$

$n = 2$ $\quad x_o(2) = \frac{x(2) - x(-2)}{2} = 2/2 = 1$ $\quad x_o(n) = \{-1, -1, \underset{\uparrow}{0}, 1, 1\}$

(5) find the even & odd components of the signal :-

$$x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$$

$$x_e(n) = \{1/2, 0, \underset{\uparrow}{2}, 0, 1/2\}$$

$$x_o(n) = \{-5/2, 1, \underset{\uparrow}{0}, -1, 5/2\}$$

(6) $x(n) = \{\underset{\uparrow}{1}, 0, -1, 2, 3\}$

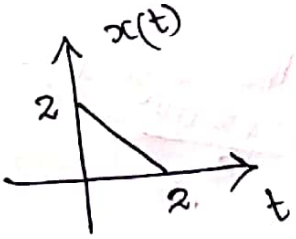
$x(0) = 1,\ x(1) = 0,\ x(2) = -1,\ x(3) = 2,\ x(4) = 3$

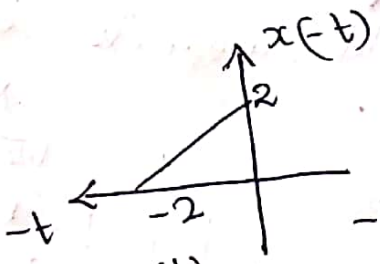
$$x_e(n) = \{\underset{\uparrow}{1}, 0, -1/2, 1, 3/2\}$$

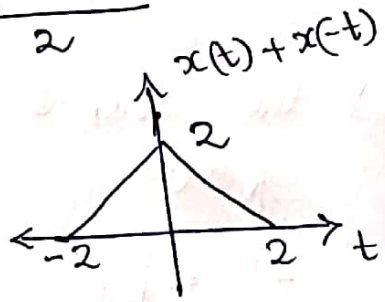
$$x_o(n) = \{\underset{\uparrow}{0}, 0, -1/2, 1, 3/2\}$$

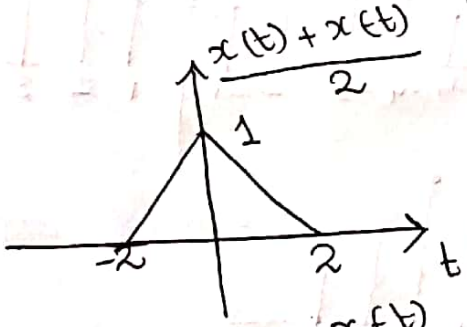
(7) Find the even & odd components of the signal :-

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

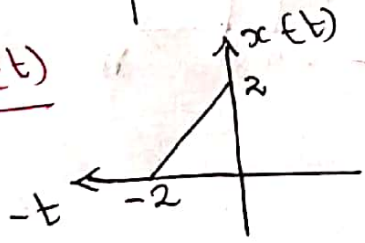


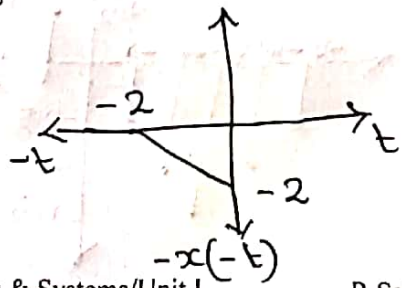


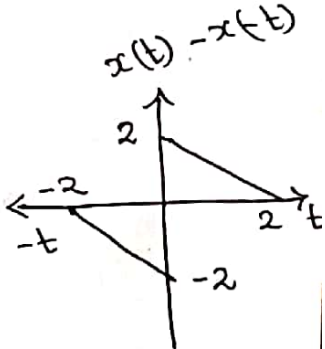


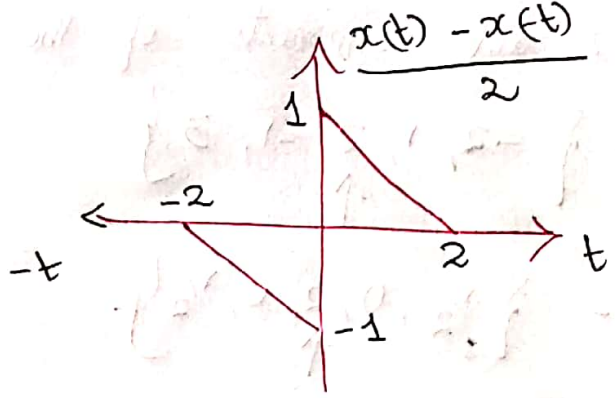


$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

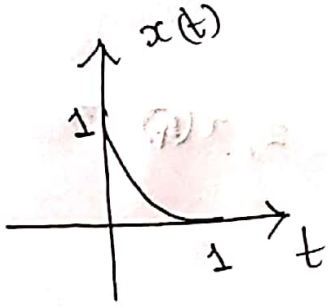


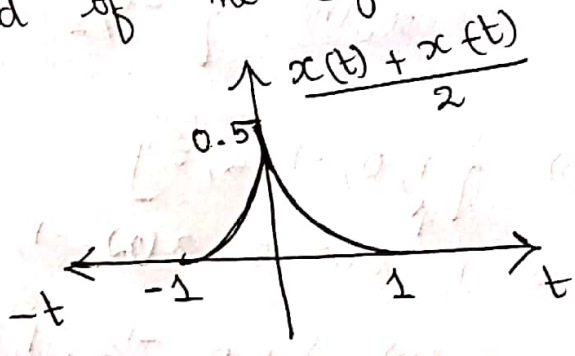


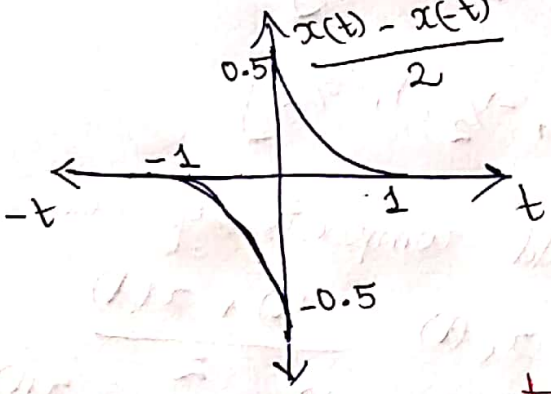




(8) Find the even & odd of the signal :-







(9) Find the even & odd components of the signal :-