Converse, contrapositive and Inverse proposition:

Defn.:

If $P \to Q$, then

$Q \to P$ is called its converse

$\neg Q \to \neg P$ is called its contrapositive

$\neg P \to \neg Q$ is called its Inverse.

Remarks:

i). The conditional proposition and its contrapositive are logically equivalent. ie, $(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

ii). The conditional proposition and its converse are not logically equivalent. ie, $(P \to Q) \not\Leftrightarrow (Q \to P)$

Example:

1). Obtain converse, contrapositive and inverse for the statement "Team India wins whenever Dhoni is a captain"

Now, P : Dhoni is a captain

Q : Team India wins

$P \to Q$: If Dhoni is a captain, then Team India wins. (conditional)

$Q \to P$: If team India wins then dhoni is a captain. (converse)

$\neg Q \to \neg P$: If ~~the~~ crops dos team India does not wins then dhoni is not a captain. (contra positive)

$\neg P \to \neg Q$: If Dhoni is not a captain then team India does not wins.

2). Obtain "If ~~it~~ it rains then the crops will grow."

P : It rains

Q : The crops will grow.

$P \to Q$: If it rains then the crops will grow

$Q \to P$: If the crops will grow then it rains

$\neg Q \to \neg P$: If the crops will not grow then it does not rains

$\neg P \to \neg Q$: If it does not rains then the crops will not grow.

Other Connectives:

(i) NAND → a combination of NOT & AND denoted by ↑

(ii) NOR → a combination of NOT & OR denoted by ↓

which is defined as

$P \uparrow Q = \neg(P \wedge Q)$ and $P \downarrow Q = \neg(P \vee Q)$

Normal forms:

The statement written in the standard form interms of $\vee$, $\wedge$ and $\neg$ then it is called the normal forms.

Note: (i) Conjunction ($\wedge$) is denoted as product.

(ii) Disjunction ($\vee$) is denoted as sum.

Elementary product:

A pdt. of the variables and their negations in a formula is called an elementary product.

Eg: P, $\neg P \wedge Q$, $\neg Q \wedge P$, $P \wedge \neg P$, $Q \wedge \neg P$

Elementary sum:

A sum of the variables and their negations in a formula is called an elementary sum.

Eg: P, $\neg P \vee Q$, $\neg Q \vee P$, $P \vee \neg P$, $Q \vee \neg P$.

Disjunctive Normal form (DNF)

A statement formula which is equivalent to a given formula and which consists of a sum of elementary products is called a Disjunctive normal form of the given formula.

$$DNF = \begin{pmatrix}\text{Elementary}\\ \text{Product}\end{pmatrix} \vee \begin{pmatrix}\text{Elementary}\\ \text{Product}\end{pmatrix} \vee \ldots \vee \begin{pmatrix}\text{Elementary}\\ \text{product}\end{pmatrix}$$

Conjunctive Normal form:

A statement formula which is equivalent to a given formula and which consists of a product of elementary sum is called a conjunctive normal form.

$$CNF = \begin{pmatrix}\text{Elementary}\\ \text{sum}\end{pmatrix} \wedge \begin{pmatrix}\text{Elementary}\\ \text{sum}\end{pmatrix} \wedge \ldots \wedge \begin{pmatrix}\text{Elementary}\\ \text{sum}\end{pmatrix}.$$

Obtain the DNF and CNF of the formula

$P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$

DNF:

$P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$

$\Leftrightarrow \neg P \vee [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$ material Implication law

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge \neg(\neg Q \vee \neg P)]$ material implication law

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge (Q \wedge P)]$ Demorgan's law

$\Leftrightarrow \neg P \vee [(\neg P \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))]$ Distributive law

$\Leftrightarrow \neg P \vee [\neg P \wedge (Q \wedge P)] \vee [(Q \wedge Q) \wedge P]$ Associative law

$\Leftrightarrow \neg P \vee [\neg P \wedge (Q \wedge P)] \vee [Q \wedge P]$ Idempotent law

CNF:

$P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$ material implication law

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge \neg(\neg(Q \wedge P))]$ Demorgan's law

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge (Q \wedge P)]$ Double negation law

$\Leftrightarrow [\neg P \vee (\neg P \vee Q)] \wedge [\neg P \vee (Q \wedge P)]$ Distributive law

$\Leftrightarrow [\neg P \vee Q] \wedge [\neg P \vee (Q \wedge P)]$ Idempotent law

$\Leftrightarrow (\neg P \vee Q) \wedge [(\neg P \vee Q) \wedge (\neg P \vee P)]$ Distributive law

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee Q) \wedge (\neg P \vee P)$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee P)$

2]. Obtain a DNF of $P \wedge (P \to Q)$

Now $P \wedge (P \to Q) \Leftrightarrow P \wedge (\neg P \vee Q)$

$\Leftrightarrow (P \wedge \neg P) \vee (P \wedge Q)$ Distributive law

Since the given statement formula is written interms of sum of elementary products.