



UNIT 2 FOURIER SERIES
ODD AND EVEN FUNCTIONS

Find the Fourier series for

$$* \quad f(x) = \begin{cases} L+x & , -L < x < 0 \\ L-x & , 0 < x < L \end{cases}$$

$$\phi_1(x) = L+x$$

$$\phi_1(-x) = L-x = \phi_2(x)$$

$$\phi_2(x) = L-x$$

$$\phi_2(-x) = L+x = \phi_1(x)$$

$\therefore f(x)$ is even function.

\therefore The Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx = \frac{2}{L} \int_0^L f(x) dx$$

$$= \frac{2}{L} \int_0^L (L-x) dx$$

$$= \frac{2}{L} \left[Lx - \frac{x^2}{2} \right]_0^L = \frac{2}{L} \left[L^2 - \frac{L^2}{2} \right]$$

$$= \frac{2}{L} \left[\frac{L^2}{2} \right] = L$$

$$\boxed{a_0 = L}$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L (L-x) \cos\left(\frac{n\pi x}{L}\right) dx$$



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$$u = L - x$$

$$u' = -1$$

$$u'' = 0$$

$$v = \cos\left(\frac{n\pi x}{L}\right)$$

$$v_1 = \frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)}$$

$$v_2 = \frac{-\cos\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)^2}$$

$$= \frac{2}{L} \left[(L-x) \frac{\sin\left(\frac{n\pi x}{L}\right)}{\frac{n\pi}{L}} - (-1) \left(\frac{-\cos\left(\frac{n\pi x}{L}\right)}{\frac{n^2\pi^2}{L^2}} \right) \right]_0^L$$

$$= \frac{2}{L} \left[0 - \frac{\cos\left(\frac{n\pi L}{L}\right)}{\frac{n^2\pi^2}{L^2}} - 0 + \frac{-\cos 0}{\frac{n^2\pi^2}{L^2}} \right]$$

$$= \frac{2}{L} \cdot \frac{L^2}{\pi^2 n^2} [1 - \cos n\pi]$$

$$= \frac{2L}{n^2\pi^2} [-(-1)^n + 1]$$

$$\therefore f(x) = \frac{L}{2} + \sum_{n=1}^{\infty} \frac{2L}{n^2\pi^2} [1 - (-1)^n] \cos\left(\frac{n\pi x}{L}\right)$$

$$= \frac{L}{2} + \frac{2L}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (1 - (-1)^n) \cos\left(\frac{n\pi x}{L}\right)$$