predicate calculus

A declarative sentence contains subject and predicate.

predicate

A part of a declarative sentence describing the properties of an object (or) relation among object is called a predicate.

Eg:

1. Tiger is an wild animal. It is denoted by $P(S)$.

$\underbrace{\text{Tiger}}_{S}$ is $\underbrace{\text{an wild animal}}_{P}$

2. Sam is poor and Ram is intelligent

It is denoted by $P(S) \wedge I(r)$

Quantifier:

Quantifier is the one which is used to quantify the nature of variables.

Types of quantifier:

1. Universal Quantifier $(\forall x)$ or (x)

The quantifier "for all x" is called universal quantifier.

Eg: 1 For all x, x is an integer

In symbolic form, $\forall x, I(x)$

2. Every apple is red.

For all x, if x is an apple then x is red.

$(\forall x)[A(x) \to R(x)]$

2. Existential quantifier: $(\exists x)$ "for some x" is called the quantifier the existential quantifier.

Eg: Some men are intelligent
There exist an x such that x is a man and x is intelligent.

$(\exists x)(M(x) \wedge I(x))$

Bound and free variables:

The variable is said to be bound if it is concerned with either universal $(\forall x)$ or existential $(\exists x)$ quantifier.

Otherwise it is called free variable.

Scope of the quantifier is the formula following the quantifier

Eg: $(x)\ P(x, y) \Rightarrow x$ is bound variable
y is free variable
$P(x, y)$ is the scope of the quantifier.

Theory of Inference for Predicate calculus

1. Universal Specification [US Rule]

$(\forall x)\ P(x) \Rightarrow P(y)$

2. Universal Generalization [UG Rule]

$P(y) \Rightarrow (\forall x)\ P(x)$

3. Existential Specification [ES Rule]

$(\exists x)\ P(x) \Rightarrow P(y)$

4. Existential Generalization [EG Rule]

$P(y) \Rightarrow (\exists x)\ P(x)$

1. Show that $(\exists x)\, M(x)$ follows logically
$(x)\, [H(x) \rightarrow M(x)]$, $(\exists x)\, H(x)$

	Step	premises	Rule
	1.	$(x)\,[H(x) \rightarrow M(x)]$	P
{1}	2.	$H(y) \rightarrow M(y)$	US
	3.	$(\exists x)\, H(x)$	P
{3}	4.	$H(y)$	ES
{2,4}	5.	$M(y)$	T $P, P\rightarrow Q \Rightarrow Q$
	6.	$(\exists x)\, M(x)$	EG

2. All humans are mortal. Sachin is a human
Therefore he is mortal.

$H(x)$: x is a human
$M(x)$: x is Mortal
$H(S)$: Sachin is a human

The premises are.
$(\forall x)\,[H(x) \rightarrow M(x)]$, $H(S)$
conclusion: $M(S)$

	Step	Premises	Rule
	1.	$(\forall x)\,[H(x) \rightarrow M(x)]$	P
{1}	2	$H(S) \rightarrow M(S)$	US
	3.	$H(S)$	P
{2,3}	4.	$M(S)$	T $P, P\rightarrow Q \Rightarrow Q$

3. Show that the premises, "one student in this class knows how to write programs in JAVA" & "Everyone who knows how to write program in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job".

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Let $A(x)$: x is in this class

$J(x)$: x knows how to write program in JAVA

$H(x)$: x can get a high paying job.

The premises are,

$(\exists x)\,(A(x) \wedge J(x))$, $(\forall x)\,(J(x) \rightarrow H(x))$

Conclusion: $\exists x\,(A(x) \wedge H(x))$

	Step	premises	Rule
	1.	$(\exists x)(A(x) \wedge J(x))$	P
{1}	2.	$A(y) \wedge J(y)$	ES
{2}	3.	$A(y)$	T $A(y) \wedge J(y) \Rightarrow A(y)$
{2}	4.	$J(y)$	T $A(y) \wedge J(y) \Rightarrow J(y)$
	5.	$(\forall x)\,(J(x) \rightarrow H(x))$	P
{5}	6.	$J(y) \rightarrow H(y)$	US
	7.	$H(y)$	T
{3,7}	8.	$A(y) \wedge H(y)$	T $A(y), H(y) \Rightarrow A(y) \wedge H(y)$
	9.	$(\exists x)\,(A(x) \wedge H(x))$	EG

4]. Verify the validity of the following argument. "Every living thing is a plant or an animal" "John's gold fish is alive and it is not a plant" "All animals have hearts". Therefore, "John's gold fish has a heart".

$L(x)$: x is a living thing $\quad L(j)$: j is alive

$P(x)$: x is a plant $\quad P(j)$: j is not a plant

$A(x)$: x is an animal $\quad H(j)$: j has a heart

$H(x)$: x is a heart

Given: $(\forall x)\,[L(x) \rightarrow P(x) \vee A(x)]$

$L(j) \wedge \neg P(j)$, $(\forall x)\,[A(x) \rightarrow H(x)]$

Conclusion: $H(j)$

Step		Premises	Rule
	1.	$(\forall x)\,[L(x) \rightarrow P(x) \vee A(x)]$	P
$\{1\}$	2.	$L(j) \rightarrow P(j) \vee A(j)$	US
	3.	$L(j) \wedge \neg P(j)$	P
$\{3\}$	4.	$L(j)$	T $P \wedge Q \Rightarrow P$
$\{2,4\}$	5.	$P(j) \vee A(j)$	T $P, P \rightarrow Q \Rightarrow Q$
$\{5\}$	6.	$\neg P(j) \rightarrow A(j)$	T $P \rightarrow Q \Leftrightarrow \neg P \vee Q$
	7.	$(\forall x)\,[A(x) \rightarrow H(x)]$	P
$\{8\}$	8.	$A(j) \rightarrow H(j)$	US
	9.	$\neg P(j) \rightarrow H(j)$	T
$\{3\}$	10.	$\neg P(j)$	T $P \wedge Q \Rightarrow P, Q$
	11.	$H(j)$	T $P, P \rightarrow Q \Rightarrow Q$

5]. "All rock music is loud music", "Some rock music exist" Therefore "Some loud music exist"

$R(x)$: x is a rock music
$L(x)$: x is a loud music

Given. $(\forall x)\,[R(x) \rightarrow L(x)]$, $(\exists x)\,R(x)$

Conclusion: $(\exists x)\,L(x)$

Step		Premises	Rule
	1.	$(\forall x)\,(R(x) \rightarrow L(x))$	P
$\{1\}$	2.	$R(y) \rightarrow L(y)$	US
	3.	$(\exists x)\,R(x)$	P
$\{3\}$	4.	$R(y)$	ES
$\{2,4\}$	5.	$L(y)$	T
	6.	$(\exists x)\,L(x)$	EG

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6] Establish the validity of argument.
i). All integers are rational nos.
ii) Some integers are power of three.
iii) Therefore some rational numbers are power of 3.

$I(x)$: x is an integer
$R(x)$: x is an rational number
$P(x)$: x is power of 3.
Soln. $(\forall x)(I(x) \rightarrow R(x))$, $(\exists x)(I(x) \wedge P(x))$
conclusion : $(\exists x)[R(x) \wedge P(x)]$

Step	Premises	Rule
1.	$(\forall x)[I(x) \rightarrow R(x)]$	P
{1} 2.	$I(y) \rightarrow R(y)$	US
3	$(\exists x)[I(x) \wedge P(x)]$	P
{3} 4.	$I(y) \wedge P(y)$	ES
{4} 5.	$I(y)$	T $P \wedge Q \Rightarrow P$
{2,5} 6.	$R(y)$	T $P, P \rightarrow Q \Rightarrow Q$
{4} 7.	$P(y)$	T $P \wedge Q \Rightarrow Q$
{6,7} 8.	$R(y) \wedge P(y)$	T $P, Q \Rightarrow P \wedge Q$
9.	$(\exists x)[R(x) \wedge P(x)]$	EG

7]. Show that $(x)[P(x) \vee Q(x)] \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ by indirect proof.

Premises : $(x)[P(x) \vee Q(x)]$
Conclusion : $(x)P(x) \vee (\exists x)Q(x)$

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Step	Premises	Rule
1.	$(x)[P(x) \vee Q(x)]$	P
{1} 2.	$P(y) \vee Q(y)$	US
3.	$\neg[(x)P(x) \vee (\exists x)Q(x)]$	Negation of conclusion
{3} 4.	$(\exists x)\neg P(x) \wedge (x)\neg Q(x)$	T $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
{4} 5.	$(\exists x)\neg P(x)$	T $P \wedge Q \Rightarrow P$
{5} 6.	$\neg P(y)$	ES
7.	$(x)\neg Q(x)$	T $P \wedge Q \Rightarrow Q$
{7} 8.	$\neg Q(y)$	US
{6,8} 9.	$\neg P(y) \wedge \neg Q(y)$	T $P, Q \Rightarrow P \wedge Q$
{9} 10.	$\neg(P(y) \vee Q(y))$	T $\neg P \wedge \neg Q \Leftrightarrow \neg(P \vee Q)$
{9,10} 11.	$[P(y) \vee Q(y)] \wedge \neg[P(y) \vee Q(y)]$	T $P, Q \Rightarrow P \wedge Q$
{11} 12.	F	T $P \wedge \neg P \Leftrightarrow F$

8) Using CP rule, obtain the following implication $(\forall x)[P(x) \rightarrow Q(x)], (\forall x)[R(x) \rightarrow \neg Q(x)] \Rightarrow$ ~~$R(y) \rightarrow \neg P(y)(x)$~~ $(x)[R(x) \rightarrow \neg P(x)]$

Step	Premises	Rule
1.	$(\forall x)[P(x) \rightarrow Q(x)]$	P
2.	$(\forall x)[R(x) \rightarrow \neg Q(x)]$	P
{2} 3	$R(y) \rightarrow \neg Q(y)$	US
4	$R(y)$	P (assumed)
{3,4} 5.	$\neg Q(y)$	T $P, P \rightarrow Q \Rightarrow Q$
{1} 6.	$P(y) \rightarrow Q(y)$	US
{5,6} 7.	$\neg P(y)$	T $P \rightarrow Q, \neg Q \Rightarrow \neg P$

{4,7}	8.	$R(y) \to \neg P(y)$	CP
{8}	9.	$(\forall x)\,[R(x) \to \neg P(x)]$	UG