HARMONIC ANALYSIS :

The process of finding the Fourier series for a function given by numerical value is known as harmonic analysis.

WKT, the Fourier series expansion as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx .$$

In harmonic analysis, the Fourier co-efficient are given by

$$a_0 = \frac{2\Sigma y}{N} \quad ; \quad a_n = \frac{2\Sigma y \cos nx}{N} \quad ; \quad b_n = \frac{2\Sigma y \sin nx}{N}$$

Fundamental (or) First Harmonic function :

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x .$$

Second Harmonic function :

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x .$$

Third Harmonic function :

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x$$

Type I :– The value of x given interms of π.

1) Find the Fourier series expansion of period 2π for $y = f(x)$ defined in $(0, 2\pi)$ by means of the values given below :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	

Soln: Here $N = 4$.

(If the function value of I & last ordinates coincide we can omit any one of them) $\therefore N = 6$.

Let $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x$.

x	y	$y\cos x$	$y\cos 2x$	$y\cos 3x$	$y\sin x$	$y\sin 2x$	$y\sin 3x$
0	1.0	1.0	1.0	1.0	0	0	0
$\pi/3$	1.4	0.7	-0.7	-1.4	1.204	1.204	0
$2\pi/3$	1.9	-0.95	-0.95	1.9	1.634	-1.634	0
π	1.7	-1.7	1.7	-1.7	0	0	0
$4\pi/3$	1.5	-0.75	-0.75	1.5	-1.29	1.29	0
$5\pi/3$	1.2	0.6	-0.6	-1.2	-1.039	-1.039	0
Σ:	8.7	-1.1	-0.3	0.1	0.516	-0.142	0

Now $a_0 = \frac{2\Sigma y}{N} = \frac{2 \times 8.7}{6} = 2.9$

$a_1 = \frac{2\Sigma y\cos x}{N} = \frac{2 \times -1.1}{6} = -0.3667$

$a_2 = \frac{2\Sigma y\cos 2x}{N} = \frac{2 \times -0.3}{6} = -0.1$

$a_3 = \frac{2\Sigma y\cos 3x}{N} = \frac{2 \times 0.1}{6} = 0.033$

$$b_1 = \frac{2\Sigma y \sin n}{N} = \frac{2 \times 0.516}{6} = 0.1732$$

$$b_2 = \frac{2\Sigma y \sin 2n}{N} = \frac{2 \times -0.172}{6} = -0.057$$

$$b_3 = \frac{2\Sigma y \sin 3n}{N} = \frac{2 \times 0}{6} = 0$$

$\therefore f(n) = 1.45 - 0.3667 \cos n - 0.1 \cos 2n + 0.033 \cos 3n + 0.1732 \sin n - 0.057 \sin 2n$

2) Determine the first two harmonic of the Fourier Series

n:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
y:	1.98	1.30	1.05	1.30	-0.88	-0.25

Soln: Here $N = 6$

$a_0 = 1.5$

$a_1 = 0.343$

$a_2 = 0.89$

$b_1 = 1.005$

$b_2 = -0.109$.

Determine the first harmonic of the Fourier series given below:

n :	0	60	120	180	240	300	360
$y: f(n)$:	40	31	-13.7	20	3.7	-21	40

1) The following table gives the variations of a periodic function over a period T :

x:	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Soln: Here N = 6, since I & last ordinates were same.

x	y	$\theta = \frac{2\pi n}{T}$	$y\cos\theta$	$y\sin\theta$
0	1.98	0	1.98	0
T/6	1.3	π/3	0.65	1.125
T/3	1.05	2π/3	-0.525	0.909
T/2	1.3	π	-1.3	0
2T/3	-0.88	4π/3	0.44	0.762
5T/6	-0.25	5π/3	-0.125	0.216
Σ:	4.5		1.12	3.012

Now $a_0 = \frac{2 \times 4.5}{6} = 1.5$.

$$a_n = \frac{2\Sigma y\cos\theta}{6} = \frac{2 \times 1.12}{6} = 0.373$$

$$b_n = \frac{2\Sigma y\sin\theta}{6} = \frac{2 \times 3.012}{6} = 1.004$$

$\therefore f(x) = 0.75 + 0.373\cos\theta + 1.004\sin\theta$.

Find the constant term & the co-eff. of the first two sine and cosine terms in the fourier expansion of y as given in the following table.

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

Soln: Here $N = 6$. $f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi}{l}x + a_2 \cos\frac{2\pi}{l}x + b_1 \sin\frac{\pi}{l}x + b_2 \sin\frac{2\pi}{l}x$

To find l:

Here, $2l = 6 \Rightarrow l = 3$

$\therefore f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi}{3}x + a_2 \cos\frac{2\pi}{3}x + b_1 \sin\frac{\pi}{3}x + b_2 \sin\frac{2\pi}{3}x.$

x	y	$y\cos\frac{\pi}{3}x$	$y\cos\frac{2\pi}{3}x$	$y\sin\frac{\pi}{3}x$	$y\sin\frac{2\pi}{3}x$
0	9	9	9	0	0
1	18	9	−9	15.58	15.58
2	24	−12	−12	20.78	−20.78
3	28	−28	28	0	0
4	26	−13	−13	−22.51	22.51
5	20	10	−10	−17.32	−17.32
Σ:	125	−25	−7	−3.46	0

Now $a_0 = \frac{2\Sigma y}{N} = 2 \times \frac{125}{6} = 41.66$.

$a_1 = \frac{2\Sigma y \cos \frac{\pi}{3} n}{N} = \frac{2 \times -25}{6} = -8.33$.

$a_2 = \frac{2\Sigma y \cos 2\frac{\pi}{3} n}{N} = \frac{2 \times -7}{6} = -2.33$

$b_1 = \frac{2\Sigma y \sin \frac{\pi}{3} n}{N} = \frac{2 \times -3.46}{6} = -1.155$

$b_2 = \frac{2\Sigma y \sin 2\frac{\pi}{3} n}{N} = 0$.

$\therefore f(n) = 20.83 - 8.33 \cos \frac{\pi}{3} n - 2.33 \cos \frac{2\pi}{3} n - 1.155 \sin \frac{\pi}{3} n$.