Ratio and Proportion

1) **Ratio**: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as $a : b$.

In the ratio $a : b$, we call 'a' as the "first term or antecedent" and 'b', the "second term or consequent".

2) **Proportion**: The equality of two ratios is called proportion.

3) If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a & d are called extremes, while b & c are called mean terms.

4) Product of means = Product of extremes.

$a : b :: c : d \iff (b \times c) = (a \times d)$

5) **4th proportional**: If $a : b = c : d$, then d is called the 4th proportional to a, b, c

3rd proportional: If $a : b = b : c$, then c is called the 3rd proportional to a and b.

Mean Proportional: Mean proportional between a and b is $\sqrt{ab}$.

6) **Comparison Ratios**: $(a : b) > (c : d) \iff (a/b) > (c/d)$

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

7) Variation :

(i) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $(x \propto y)$.

(ii) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $\left(x \propto \left(\frac{1}{y}\right)\right)$.

8) If $\frac{a}{b} = \frac{c}{d}$, then

(i) Invertendo $\Rightarrow \frac{b}{a} = \frac{d}{c}$

(ii) Alternendo $\Rightarrow \frac{a}{c} = \frac{b}{d}$

(iii) Componendo $\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

(iv) Dividendo $\Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

(v) Componendo and dividendo

$$\Rightarrow \left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$$

(vi) If $\frac{a}{b} = \frac{c}{d} = \frac{i}{j} = \cdots = k$, then $\frac{a+c+i+\cdots}{b+d+j+\cdots} = k$

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Techniques:

1) (i) If $A : B = a : b$ & $B : C = m : n$, then
$A : B : C = am : mb : nb$ &
$A : C = am : bn$

(ii) If $A : B = a : b$, $B : C = c : d$ & $C : D = e : f$,
then $A : B : C : D = ace : bce : bde : bdf$

2) (i) If x is divided in $a : b$, then
1^{st} part $= \frac{ax}{a+b}$, 2^{nd} part $= \frac{bx}{a+b}$,

(ii) If x is divided in $a : b : c$, then
1^{st} part $= \frac{ax}{a+b+c}$, 2^{nd} part $= \frac{bx}{a+b+c}$,
3^{rd} part $= \frac{cx}{a+b+c}$.

3) The incomes of two persons are in ratio of $a : b$ and their expenditures are in the ratio of $c : d$. If each of them saves $₹ X$, then their incomes are given by $\frac{X(d-c)}{ad-bc} \times a$ & $\frac{X(d-c)}{ad-bc} \times b$, respectively
and their expenditures are given by
$\frac{X(b-a)}{ad-bc} \times c$ & $\frac{X(b-a)}{ad-bc} \times d$, respectively

4) If two numbers are in ratio $a:b$ and x is added to the numbers, then the ratio becomes $c:d$. The two numbers will be $\frac{xa(c-d)}{ad-bc}$ & $\frac{xb(c-d)}{ad-bc}$, respectively.

5. Two numbers are in ratio $a:b$ and x is subtracted from the numbers, then the ratio becomes $c:d$. The two numbers will be $\frac{xa(d-c)}{ad-bc}$ & $\frac{xb(d-c)}{ad-bc}$, respectively.

1) The ratio of $A:B = 1:3$, $B:C = 2:5$ and $C:D = 2:3$. Find the value of $A:B:C:D$.

Sol:

[Hint: Use formula 1(ii)]

Sol:

$A:B = 1:3$, $B:C = 2:5$, $C:D = 2:3$.

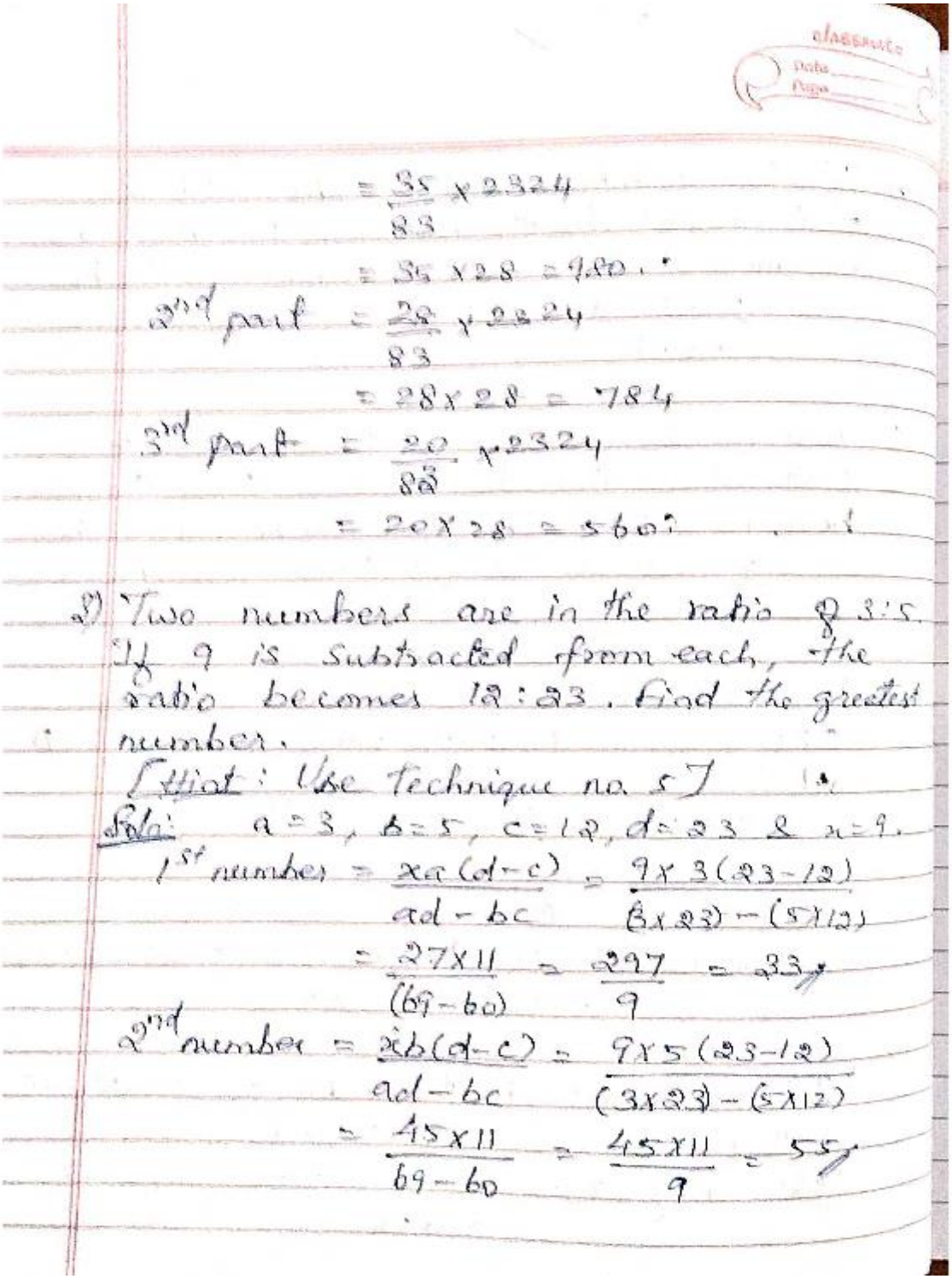
$\Rightarrow A:B:C:D = (1\times2\times2):(3\times2\times2):(3\times5\times2):(3\times5\times3)$

$= 4:12:30:45$.

2) Divide 2324 in the ratio of 35 : 28 : 20.

[Hint: Use formula 2(ii).]

Sol: 1st part $= \frac{35}{35+28+20} \times 2324$



$= \frac{35}{83} \times 2324$

$= 35 \times 28 = 980$.

2nd part $= \frac{28}{83} \times 2324$

$= 28 \times 28 = 784$

3rd part $= \frac{20}{83} \times 2324$

$= 20 \times 28 = 560$

2) Two numbers are in the ratio 3:5. If 9 is subtracted from each, the ratio becomes 12:23. Find the greatest number.

[Hint: Use Technique no 5]

Sol: $a = 3, b = 5, c = 12, d = 23$ & $x = 9$.

1st number $= \frac{xa(d-c)}{ad-bc} = \frac{9 \times 3(23-12)}{(3 \times 23)-(5 \times 12)}$

$= \frac{27 \times 11}{(69-60)} = \frac{297}{9} = 33$

2nd number $= \frac{xb(d-c)}{ad-bc} = \frac{9 \times 5(23-12)}{(3 \times 23)-(5 \times 12)}$

$= \frac{45 \times 11}{69-60} = \frac{45 \times 11}{9} = 55$