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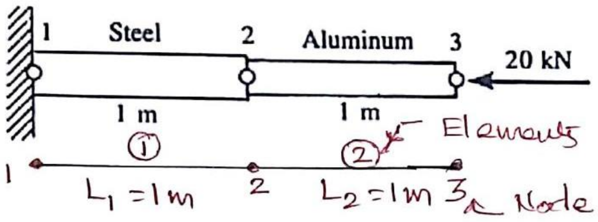
DEPARTMENT OF AEROSPACE ENGINEERING

For the bar assemblage shown in Figure 0.0, determine the nodal displacements, the forces in each element, and the reactions. Use the direct stiffness method for these problems.



$E_{st} = 200$ GPa
$A_{st} = 4 \times 10^{-4}$ m^2
$E_{al} = 70$ GPa
$A_{al} = 2 \times 10^{-4}$ m^2

Stiffness matrix For Element ①

$$k^{(1)} = \frac{A_1 E_1}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k^{(1)} = 4\times10^{-4}(m^2)\times 200\times10^{6}\left(\frac{kN}{m^2}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 800\times10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{kN}{m}$$

$$= 10^{2}\begin{bmatrix} 800 & -800 \\ -800 & 800 \end{bmatrix}\frac{kN}{m} \quad (\text{nodes } 1, 2)$$

Stiffness matrix For Element ②

$$k^{(2)} = \frac{A_2 E_2}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{2\times10^{-4}\times 70\times10^{6}}{1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k^{(2)} = 140\times10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^{2}\begin{bmatrix} 140 & -140 \\ -140 & 140 \end{bmatrix}\frac{kN}{m} \quad (\text{nodes } 2, 3)$$

Global matrix

$$k^{(G)} = 10^{2}\begin{bmatrix} 800 & -800 & 0 \\ -800 & 800+140 & -140 \\ 0 & -140 & 140 \end{bmatrix} = 10^{2}\begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix}$$

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Element Equation $\Rightarrow [K][U] = \{F\}$

global Stiff — Displacement Vector — Force Vector

Displacement Vector.

$$U = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Force Vector

$$F = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Formulation of Element Equation.

$[K][U] = [F]$

$$10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ F_3 \end{bmatrix}$$

Apply Boundary Condition. $U_1 = 0,\ F_1 = 0$
$F_2 = 0,\ F_3 = -20$

$$10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20\,kN \end{bmatrix}$$

Apply → Elimination Approach

$$10^2 \begin{bmatrix} 940 & -140 \\ -140 & 140 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -20\,kN \end{bmatrix}$$

generation of Simultaneous equation. from Eqn

$$10^2\left[940 u_2 - 140 u_3\right] = 0 \quad \cdots ①$$

$$u_3 = 6.741 u_2$$

$$10^2\left[-140 u_2 + 140 u_3\right] = -20 \quad -②$$

Substituting $u_3 = 6.741 u_2 \rightarrow ②$

$$10^2\left[-140 u_2 + 140\left[6.74 u_2\right]\right] = -20$$

$$u_2 = -0.25\times10^{-3}\,m$$

$$u_3 = -1.678\times10^{-3}\,m \quad [-1.678571429\times10^{-3}]$$

Strain Calculation

$$\xi^{(1)} = \frac{u_2 - u_1}{L_1} = \frac{-0.25\times10^{-3} - 0}{1} = -0.25\times10^{-3}$$

$$\xi^{(2)} = \frac{u_3 - u_2}{L_2} = \frac{-1.678571429\times10^{-3} + 0.25\times10^{-3}}{1}$$

$$= -1.428571429\times10^{-3}$$

Stress Calculation [By Hooke's law]

$$\sigma^{(1)} = E_1\xi^{(1)} = 200\times10^6 \times -0.25\times10^{-3} = 50000$$

$$\sigma^{(2)} = E_2\xi^{(2)} = 70\times10^6 \times -1.428571429\times10^{-3} = 10,000$$

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Theoretical Stresses for this problem are easily Calculated by

$$\sigma^{(1)}_{Theory} = \frac{P}{A_1} = \frac{20}{4\times10^{-4}} = 50{,}000$$

$$\sigma^{(2)}_{Theory} = P/A_2 = \frac{20}{2\times10^{-4}} = 10{,}000$$

Reaction force

$$[R] = [K][U] - [F]$$

$$\begin{bmatrix} R_1 \\ R_2 \to 0 \\ R_3 \to 0 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \to 0 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} f_1 \to 0 \\ F_2 \to 0 \\ F_2 \to (-20) \end{bmatrix}$$

Apply BC

$$\begin{bmatrix} R_1 \\ 0 \\ 0 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -20 \end{bmatrix}$$

$$-800\times10^2 \times u_2 - 0 = R_1$$

$$-800\times10^2 \times 1.678571429\times10^{-3} = \underline{20\ kN}$$

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