For a tapered plate of uniform thickness t=10mm as shown in Figure.1, find the displacements at the nodes by meshing into two element model. The bar has mass density $\rho = 7800 kg/m^3$, $E = 2\times10^5 MN/m^2$. In addition to self-weight, the plate is subjected to a point load P=10kN at its center. Also determine the reaction force at the support.

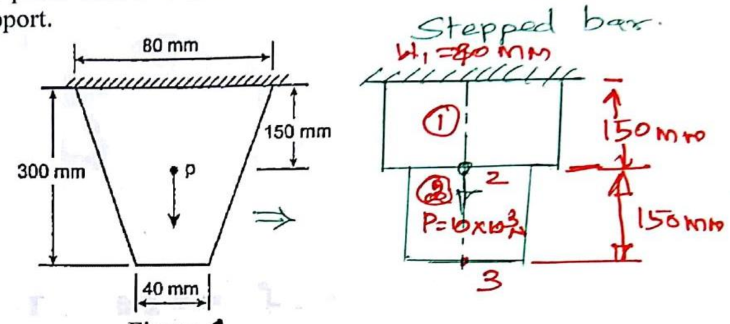


Figure .1

Solution

Area at node1, $A_1$

$= \text{Width} \times \text{thickness} = W_1 \times t_1$

$= 80\times10$

$A_1 = 800\ mm^2$

Areat at node 2, $A_2$

$$A_2 = \left[\frac{W_1 + W_3}{2}\right] \times t_2$$

$$= \left[\frac{80+40}{2}\right] \times 10$$

$A_2 = 600\ mm^2$

[$t_1, t_2, t_3 = 10\ mm$]

Area at node 3, $A_3$

$A_3 = W_3 \times t_3 = 40\times10$

$= 400\ mm^2$

Average area of element (1) $A_1^{(1)} =$

$$= \frac{\text{Area at node 1} + \text{Area at nod 2}}{2}$$

$$= \frac{800+600}{2}$$

$A_1^{(1)} = 700\ mm^2$

Average Area of element (2) $A_2^{(2)}$

$$= \frac{A_2 + A_3}{2}$$

$$= \frac{600+400}{2}$$

$A_2^{(2)} = 500\ mm^2$

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Mass density $\rho = 7800\ kg/m^3 = 7800 \times 9.81\ N/m^3$
$= 76518\ N/m^3 = 76518 \times 10^{-9}\ N/mm^3$
$= 7.6518 \times 10^{-5}\ N/mm^3$

Young's Modulus $E = 2 \times 10^5\ MN/m^2$
$= 2 \times 10^5 \times 10^6\ N/m^2$
$= 2 \times 10^5 \times 10^6 \times 10^{-6}\ N/mm^2$
$= 2 \times 10^5\ N/mm^2$

Stiffness matrix for element ①

$$K_1 = \frac{A_1^{(1)} E}{L_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{700 \times 2 \times 10^5}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 4.666 \times 2 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^5 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 9.332 & -9.332 \\ -9.332 & 9.332 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

Stiffness matrix for element ②

$$K_2 = \frac{A_2^{(2)} E}{L_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{500 \times 2 \times 10^5}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 6.666 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^5 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 6.666 & -6.666 \\ -6.666 & 6.666 \end{bmatrix} \end{array}$$

Global matrix $[K] = K_1 + K_2$

$$= 10^5 \times \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & \boxed{9.332 + 6.666} \rightarrow 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix}$$

Displacement Vector $U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

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Body force Vector of $\{F\} = \frac{\rho A L}{2}\begin{bmatrix}1\\1\end{bmatrix}$

Force Vector For element ①

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1^{(1)} L_1}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times700\times150\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}4.017\\4.017\end{bmatrix}\end{array}\begin{array}{c}1\\2\end{array}$$

Force Vector for element ②

$$\begin{bmatrix}F_2\\F_3\end{bmatrix} = \frac{\rho_2 A_2^{(2)} L_2}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times500\times150\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}2.869\\2.869\end{bmatrix}\end{array}\begin{array}{c}2\\3\end{array}$$

Global force vector

$$\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}4.017\\4.017+2.869\\2.869\end{Bmatrix} = \begin{Bmatrix}4.017\\6.886\\2.869\end{Bmatrix}$$

Assemble the finite element equation $[K]\{U\}=\{F\}$

$$10^5\begin{bmatrix}9.332 & -9.332 & 0\\-9.332 & 15.998 & -6.666\\0 & -6.666 & 6.666\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}4.017\\6.886\\2.869\end{bmatrix}$$

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Apply the boundary condition i.e at node 1 displace $u_1 = 0$.

A point load of $10 \times 10^3 N$ is acting at node 2, so add 10,000 N in $F_2$ vector

$$10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 4.017 \\ 10006.886 \\ 2.869 \end{bmatrix}$$

In the above equation $u_1 = 0$, so neglect first row and first column of [K] matrix. The reduced equation is.

$$10^5 \begin{bmatrix} 15.998 & -6.666 \\ -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 10006.886 \\ 2.869 \end{bmatrix}$$

$$[15.998 U_2 - 6.666 U_3] 10^5 = 100006.886 \quad -(1)$$

$$[-6.666 U_2 + 6.666 U_3] 10^5 = 2.869 \quad -(2)$$

Solve above equation

$u_2 = 0.01073$ mm

$u_3 = 0.01073$ mm

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Reaction force $\{R\} = [K]\{U\} - \{F\}$

$$= 10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ 0.01073 \\ 0.01073 \end{bmatrix} = \begin{bmatrix} 4.017 \\ 10{,}006.88 \\ 2.869 \end{bmatrix}$$

$$= \begin{bmatrix} -10{,}000 \\ 10{,}000 \\ 0 \end{bmatrix} - \begin{bmatrix} 4.017 \\ 10{,}006.886 \\ 2.869 \end{bmatrix}$$

$$= \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -10004.017 \\ -6.886 \\ -2.869 \end{Bmatrix} N$$

Reaction force is equivalent and opposite to applied force.

Verification $R_1 + R_2 + R_3$

$= -10004.017 - 6.886 - 2.869$

$= -10013.772\ N$

Applied force $= 4.017 + 10{,}006.886 + 2.869$

$= 10013.772\ N$

Result: Displacement $U_1 = 0$

$U_2, U_3 = 0.01073\ mm$.

Reaction force at the suppor



$R_1 = -10004.017\ N$ 5/5