



SNS COLLEGE OF TECHNOLOGY

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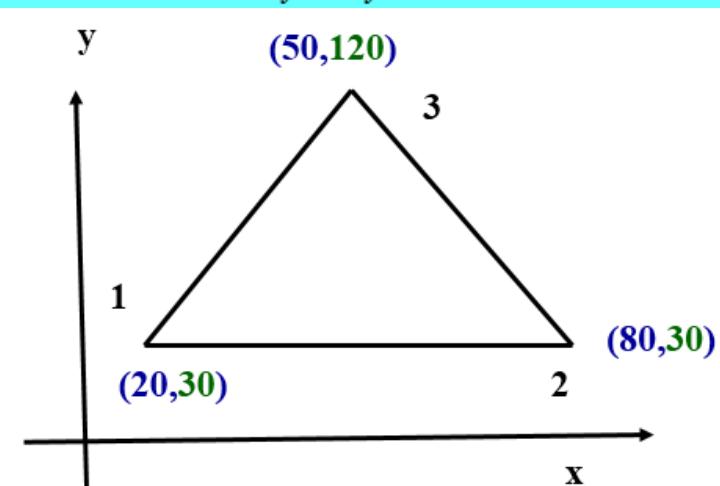
COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF AEROSPACE ENGINEERING

Evaluate the stiffness matrix for the elements shown in Figure. The coordinates are given in units of millimetres. Assume plane stress conditions. Let $E = 210GPa$, $\mu = 0.25mm$, and $t = 10 mm$. For the elements given in Figure, the nodal displacements are given as $u_1 = 2.0mm$, $v_1 = 1.0mm$, $u_2 = 0.5mm$, $v_2 = 0.0mm$, $u_3 = 3.0mm$, $v_3 = 1.0mm$. Determine the element stresses σ_x , σ_y , τ_{xy} , σ_1 , and σ_2 and the principal angle θ_p .

Finite Element Analysis



Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

- $(x_1, y_1) = (20, 30)$
- $(x_2, y_2) = (80, 30)$
- $(x_3, y_3) = (50, 120)$

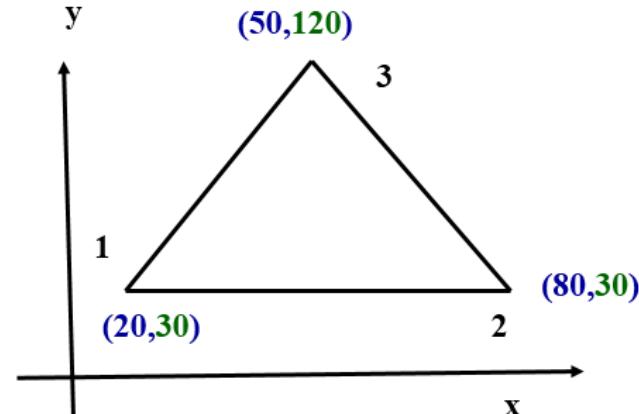
To find; $[K] = [B]^T [D] [B] \cdot A \cdot t$

Step i) Area of Triangular Element

$$A = \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = A = \frac{1}{2} \begin{bmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{bmatrix}$$

$$= \frac{1}{2} (1(80 \times 120 - 30 \times 50) - 20(1 \times 120 - 30 \times 1) + 30(1 \times 50 - 80 \times 1))$$

$$A = 2700 \text{ mm}^2$$

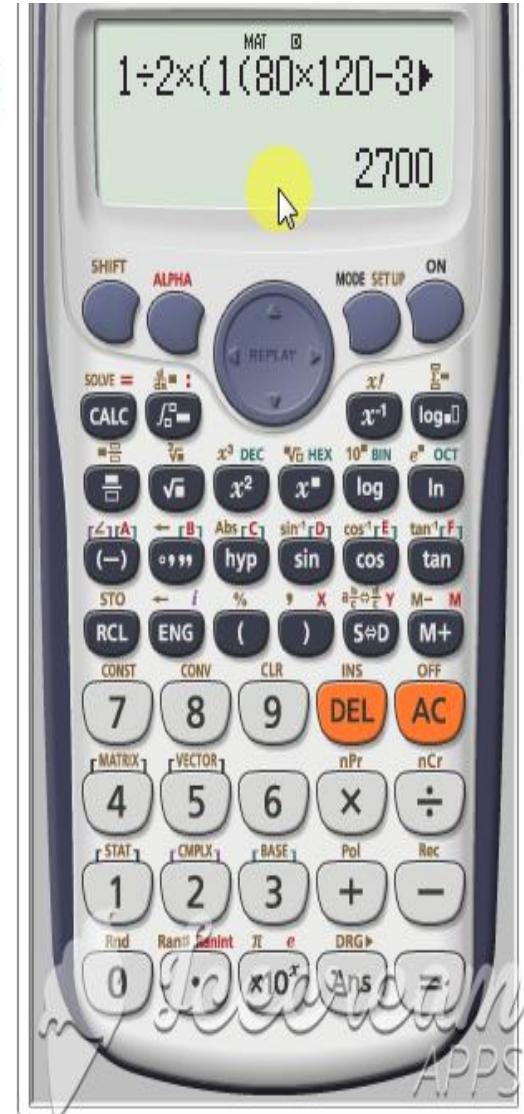


Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

$$= \frac{1}{2} (1(80 \times 120 - 30 \times 50) - 20(1 \times 120 - 30 \times 1) + 30(1 \times 50 - 80 \times 1)) \\ = 2700 \text{ mm}^2$$



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Step ii) Strain Displacement matrix

- $[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$

$$q_1 = y_2 - y_3 = 30 - 120 = -90 \quad r_1 = x_3 - x_2 = 50 - 80 = -30$$

$$q_2 = y_3 - y_1 = 120 - 30 = 90 \quad r_2 = x_1 - x_3 = 20 - 50 = -30$$

$$q_3 = y_1 - y_2 = 30 - 30 = 0 \quad r_3 = x_2 - x_1 = 80 - 20 = 60$$

$$[B] = \frac{1}{2 \times 2700} \begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

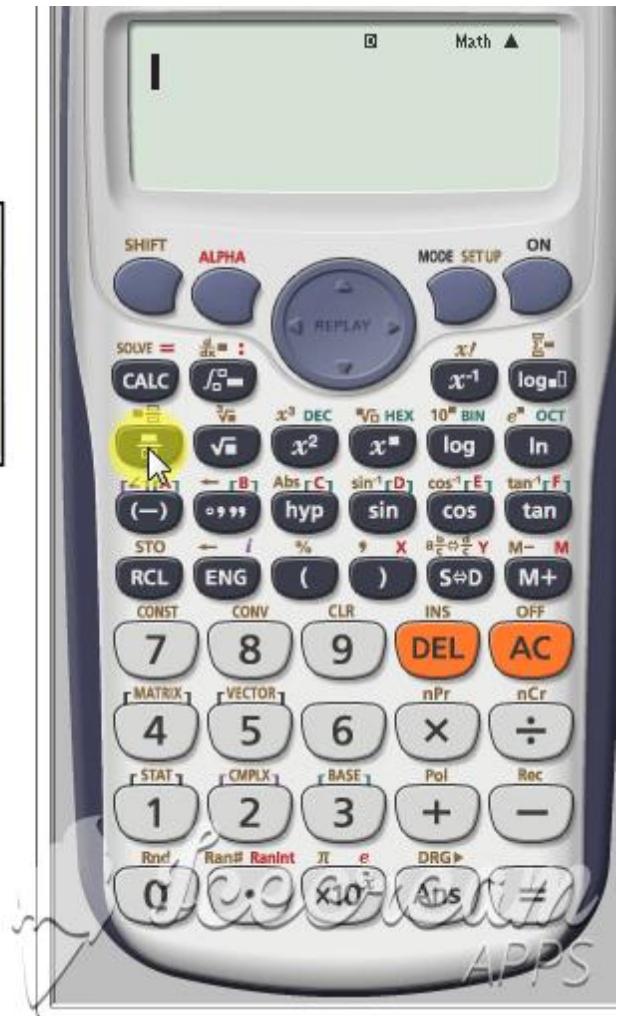
$$[B] = \frac{1}{2 \times 2700} \times [30] \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix} \quad \xrightarrow{\text{Curved arrow}} \quad [B] = \frac{1}{180} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

$$[B] = \frac{1}{2 \times 2700} \times [30] \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$



Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

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$$[B]^T = \frac{1}{180} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & 3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

ii) Stress – strain matrix $[D]$ for plane stress condition: $[D] = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$

$$[D] = \frac{2.1 \times 10^5}{1-0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$D = \frac{2.1 \times 10^5}{1-0.25^2} [0.25] \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$


$$[D] = 56000 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

Finite Element Analysis

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Constant Strain Triangular element [CST] Stiffness matrix [B]

$$\bullet [K] = [B]^T [D] [B] \cdot A \cdot t$$

$$= [D][B]$$

$$= 56 \times 10^3 \times \frac{1}{180} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$= 311.111 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

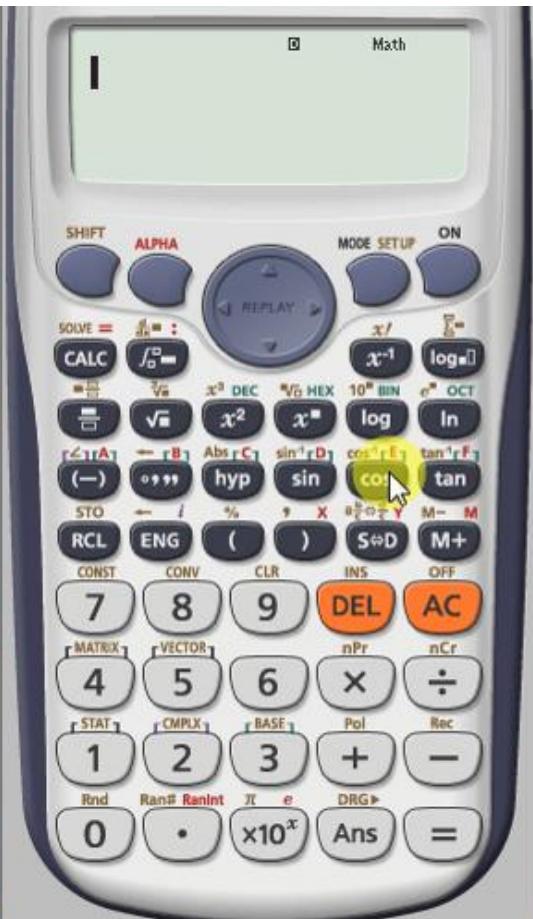
The diagram illustrates the matrix multiplication process. It shows three 3x3 matrices: Matrix A (purple), Matrix B (blue), and Matrix C (green). A red curved arrow points from Matrix A to Matrix B, indicating the first multiplication step. A blue curved arrow points from the result of the first multiplication to Matrix C, indicating the second multiplication step.

$$= 311.111 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]



$$\begin{aligned}
 & \bullet [K] = [B]^T [D] [B] \cdot A \cdot t \\
 & = [D][B] \\
 & = 56 \times 10^3 \times \frac{1}{180} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1 & 0 \\ -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ -1 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix} \\
 & \quad \text{Matrix } A[3x3] \quad \text{Matrix } B[3x3] \quad \text{Matrix } C[3x3] \\
 & = 311.111 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 \\ 0 & -1 & 0 \\ -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix} \\
 & \quad \text{Matrix } A[3x3] \times \text{Matrix } B[3x3] \quad \text{Matrix } A[3x3] \times \text{Matrix } C[3x3] \\
 & = 311.111 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}
 \end{aligned}$$

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Notes Comments

Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

To find; $[K] = [B]^T [D] [B] \cdot A \cdot t$

• $= [B]^T [D] [B]$.

$$= \frac{1}{180} \times 311.111 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$= 1.72839 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

$$\begin{aligned}
 & \text{mat } A^1 \quad \left[\begin{array}{ccc} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{array} \right] \quad \text{mat } B \quad \left[\begin{array}{cccc} -12 & -1 & 12 & -1 \\ -3 & -4 & 3 & -4 \\ -1.5 & -4.5 & -1.5 & 4.5 \end{array} \right] \quad \text{mat } C \quad \left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 2 \\ -1 & 0 & 2 & 0 & 8 \\ -4 & 0 & 8 & 0 & 0 \end{array} \right] \\
 & = 1.72839 \quad \text{mat } A^2 \quad \left[\begin{array}{c} \text{mat } A^1 \times \text{mat } B \quad \text{mat } A^1 \times \text{mat } C \\ \text{mat } A^2 \times \text{mat } B \quad \text{mat } A^2 \times \text{mat } C \end{array} \right]
 \end{aligned}$$

$$= A \cdot t[B]^T[D][B]$$

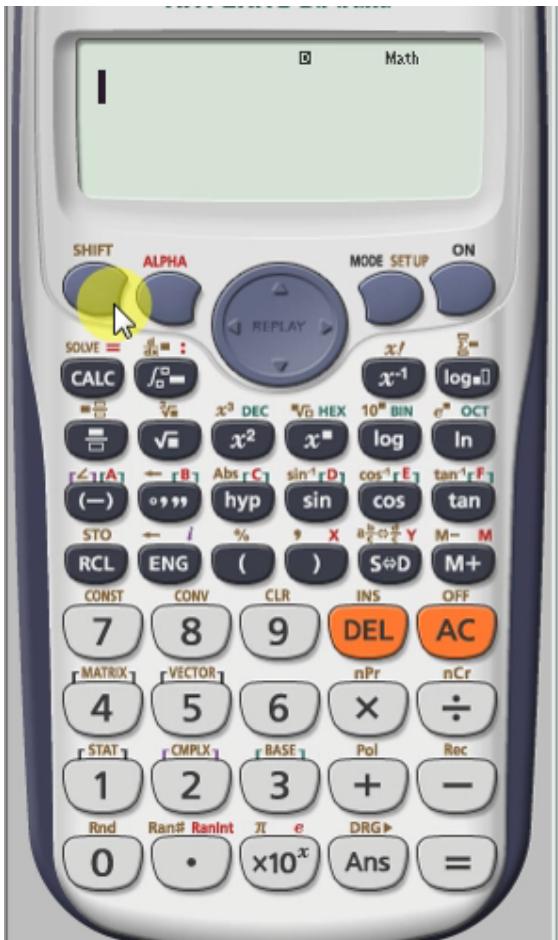
$$= 46666.53 \left[\begin{array}{ccccccc} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{array} \right]$$



Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]



Constant Strain Triangular element [CST] Stiffness matrix [B]

$$= 1.72839 \begin{matrix} mat\ A^1 \\ \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \end{matrix} \begin{matrix} mat\ B \\ \begin{bmatrix} -12 & -1 & 12 \\ -3 & -4 & 3 \\ -1.5 & -4.5 & -1.5 \end{bmatrix} \end{matrix} \begin{matrix} mat\ C \\ \begin{bmatrix} -1 & 0 & 2 \\ -4 & 0 & 8 \\ 4.5 & 3 & 0 \end{bmatrix} \end{matrix}$$
$$= A.t[B]^T[D][B] \quad \begin{matrix} [mat\ A^1 \times mat\ B & mat\ A^1 \times mat\ C] \\ [mat\ A^2 \times mat\ B & mat\ A^2 \times mat\ C] \end{matrix}$$

$$= 46666.53 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

Prepared by Dr.M.SUBRAMANIAN/Professor/Mechanical/ Finite Element Analysis

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Finite Element Analysis

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Constant Strain Triangular element [CST] Stiffness matrix [B]



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$$= 1.72839 \begin{matrix} mat A^1 \\ \left[\begin{array}{ccc} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{array} \right] \\ mat A^2 \end{matrix} \begin{matrix} mat B \\ \left[\begin{array}{ccc} -12 & -1 & 12 \\ -3 & -4 & 3 \\ -1.5 & -4.5 & -1.5 \end{array} \right] \\ mat A^1 \times mat B \end{matrix} \begin{matrix} mat C \\ \left[\begin{array}{ccc} -1 & 0 & 2 \\ -4 & 0 & 8 \\ 4.5 & 3 & 0 \end{array} \right] \\ mat A^1 \times mat C \end{matrix}$$

$$= A \cdot t[B]^T[D][B] = 46666.53 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

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APPS

Finite Element Analysis

Two-Dimensional Scalar Variable Problems

Constant Strain Triangular element [CST] Stiffness matrix [B]

THANK YOU