

# SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution) Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp; Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT) COIMBATORE-641 035, TAMIL NADU

#### **DEPARTMENT OF MATHEMATICS**

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Subgroups:
2082.
 let (G1, *) be a group. Then (H, *) & said to be
Subgroup of (G, *) & HCG and (H, *) Uself 98
a group under the operation &.
  te, (H, *) is said to be a subgroup of (6, *) &
   i). 88H
  ii). For any ach, atch
  ii). For a, bEH, a*bEH
 Theostern: 1
 The necessary and sufficient condition that
 non empty subset H of a group of to be a
 Subgroup 98 a, DEH > a * b TEH.
  Proof:
  Necessary condition:
  A98 wore that H B a Subgroup of G1.
  TO PROVE a * b + EH
  Let a, b \in H \Rightarrow b^{T} \in H (IDVOY3e)
   Then a*b'EH
   Sufficient condition:
   Assume that a, b \in H \Rightarrow a * b^{-1} \in H
  TO PSEOVE H B a subgroup of GI.
  i). closure:
   Let a, bEH
    Since DEH > DEH
   Let a, b^{-1} \in H \Rightarrow a * (b^{-1})^{-1} \in H by (1)
                      axbEH => H & closed
   ii) Identity :
      Let a \in H \Rightarrow a^{-1} \in H
      Then axaTEH
                    [GI-> group]
               eEH
           7
   Hence the 9 dentify elt. CEH.
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| iii). Inverse:<br>Let Q, QEH  |
|---|
| $\Rightarrow e * a^{-1} \in H$ by (1)                                 |
| $a^{-1} \in H$  |
| Hence H B a subgroup of G Sence H Uself                               |
| 15 0 620 0  |
| b a group.  |
| Theorem: 2  |
| The Phtersection of & Subgroups of a                                  |
| group is also a subgroup of the group.                                |
| (07) H. and H.  |
| Let G be a group and H, and Ha are                                    |
| Subgroups of G. Then H, nHz is also a                                 |
| Sabgroup of Gr.<br>Proof:   |
| Proof:<br>Let H, and Ha be the two subgroups of G.                    |
| To prove HINH& is a Subgroup of GI.                                   |
|   |
| H, n Hz + & [" at least the 9d entity elt. B<br>present 9n H1 and Hz] |
| Let a, bE HINHZ   |
| $\Rightarrow a, b \in H, and a, b \in H_{2}$                          |
|   |
| => a+b+EH, and a+b+EHz [sence H, and Hz core                          |
| $\Rightarrow a * b^{\dagger} E H, n H_{a}$ Subgroups]                 |
| FOR a, b E H, n Ha, we've a* bt E H, n Ha.                            |
| .". HINHa is a subgroup [By above Theorem ].                          |
| Theoriem: 3   |
| The unton of two subgroups of a group                                 |
| nord not be a subgroup.   |



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Theosem: 4
   The unton of two subgroups of a group or B
a subgroup 966 one is contained in the other.
Proof:
 ABBUME H and K are two subgroups of GI
 and HSKORKSH.
 · HUK=K OR HUK=H
  Hence HUK is a subgroup.
 Convensely,
     Suppose HUK is a subgroup of Gr.
TO PHOVE HCK OR KCH.
Suppose that H9s not contained int and K is not
Contained 90 H.
Then Jells. a, b 7 aEH and a EK -> (1)
                 bEK and bitH ->(2)
  Clearly, a, bEHUK RADCE HUN is a subgroup of G
                          OBE HUK
     Hence a + b E H OA a + b E H
 case 1). Let axb EH
   Since, a EH => a-1 EH
  Honce at * (a*b) EH
       (at *a) * DEH ASSociative
           e*bEH
           bEH which is a contradiction to ocor
                                assumption
Case 2). Leta +b E K
   SANCE DEK => 5 EK
  Hence 5 * (a*b) EK
    5'* (b*a) EK
   (5'+b)+aEK
                           whitch is a > = toown
            e*aEK⇒ aEK
                                 aggumption.
      . Over assumption is wrong.
```