



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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## DEPARTMENT OF MATHEMATICS

Lattice:

A lattice is a partially ordered set  $(L, \leq)$  in which every pair of elements  $a, b \in L$  has both LUB and GLB.

Note:

LUB  $\{a, b\} = a \vee b$  (or)  $a + b$  (or)  $a \oplus b$  (a join b)

GLB  $\{a, b\} = a \wedge b$  (or)  $a \cdot b$  (or)  $a * b$  (a meet b)

A lattice is denoted by triplet  $(L, \wedge, \vee)$  (or)  $(L, \cdot, +)$

Example:

1. Let  $A$  be any finite set

Then  $(P(A), \subseteq)$  is a lattice

$\wedge \rightarrow$  union

$\vee \rightarrow$  intersection

Problems:

1. Determine whether the posets

i.  $(\{1, 2, 3, 4, 5\}, \mid)$

ii.  $(\{1, 2, 4, 8, 16\}, \mid)$  are

lattices.

Soln.

i).  $R = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 4)\}$

Hasse diagram:

UB  $\{2, 3\} =$  does not exist

\*  $\therefore$

LUB  $\{2, 3\} =$  does not exist

UB  $\{1, 2\} = \{2, 4\}$

LUB  $\{1, 2\} = 2$

Here LUB  $\{2, 3\}$  does not exist.

$\therefore$  It is not a Lattice.



## DEPARTMENT OF MATHEMATICS

1).  $R = \{ (1, 2), (1, 4), (1, 8), (1, 16), (2, 4), (2, 8), (2, 16), (4, 8), (4, 16), (8, 16) \}$



Every pair of elt. have both GLB and LUB.  
 $\therefore I \cong$  a lattice.

2).  $(\mathbb{Z}^+, /)$  is a lattice

Soln.

Let  $a, b \in \mathbb{Z}^+$

$$\text{LUB}\{a, b\} = \text{LCM}\{a, b\}$$

$$\text{GLB}\{a, b\} = \text{GCD}\{a, b\}$$

For eg.,  $a = 4, b = 20$

$$\text{LUB}\{4, 20\} = \text{LCM}\{4, 20\} = 20$$

$$\text{GLB}\{4, 20\} = \text{GCD}\{4, 20\} = 4$$

3). Draw Hasse diagram of all lattice with upto five elt.

Soln.

