Lattice :

A lattice is a partially ordered set $(L, \leq)$ in which every pair of elements $a, b \in L$ ~~has a~~ have both LUB and GLB.

Note :

LUB $\{a, b\} = a \vee b$ (or) $a + b$ (or) $a \oplus b$ (a joint b)

GLB $\{a, b\} = a \wedge b$ (or) $a \cdot b$ (or) $a * b$ (a meet b)

A lattice is denoted by triplet $(L, *, \oplus)$ (or) $(L, \wedge, \vee)$ (or) $(L, \cdot, +)$

Example :

1. Let A be any finite set

Then $(P(A), \subseteq)$ is a Lattice

$\wedge \rightarrow$ union

$\vee \rightarrow$ intersection

Problems :

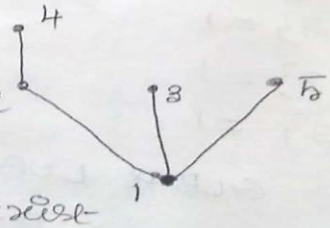
1. Determine whether the posets

i). $(\{1, 2, 3, 4, 5\}, /)$ ii). $(\{1, 2, 4, 8, 16\}, /)$ are lattices.

Soln.

i). $R = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 4)\}$

Hasse diagram :



LUB $(2, 3)$ = ~~does~~ does not exist

LUB $\{2, 3\}$ = does not exist

UB $\{1, 2\} = \{2, 4\}$

LUB $\{1, 2\} = 2$

Here LUB $\{2, 3\}$ does not exist.

$\therefore$ It is not a Lattice.

ii). $R = \{(1,2), (1,4), (1,8), (1,16), (2,4), (2,8), (2,16), (4,8), (4,16), (8,16)\}$

Every pair of elts have both GLB and LUB.
∴ It is a lattice.

2]. $(Z^+, /)$ is a lattice

Soln.

Let $a, b \in Z^+$

$LUB\{a,b\} = LCM\{a,b\}$

$GLB\{a,b\} = GCD\{a,b\}$

For eg., $a = 4, b = 20$

$LUB\{4,20\} = LCM\{4,20\} = 20$

$GLB\{4,20\} = GCD\{4,20\} = 4$

3]. Draw Hasse diagram of all lattice with upto five elt.

Soln.