SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE – 35

23MAT201 –PARTIAL DIFFERENTIAL EQUATIONS AND TRANSFORMS

UNIT – II

FOURIER SERIES

PART A

TWO MARKS

1. Explain periodic function with two examples.

Solution: A function f(x) is said to have a period T if for all x, $f(x+T) = f(x)$,

Where T is a positive constant. The least value of $T > 0$ *is called the period of f(x).*

For examples, $f(x) = \text{Sin}x$

$f(x + 2\pi) = \text{Sin}(x + 2\pi) = \text{Sin}x$

Here, $f(x) = f(x + 2\pi)$

2. State Dirichlet's condition for a given function to expend in Fourier series.

Solution: Any function f(x) can be developed as a Fourier series, provided

i) f(x) is periodic, single valued & finite.
ii) f(x) has a finite number of discontinuities in any one period
iii) f(x) has a finite number of maxima and minima

3. State general Fourier series.

solution: The Fourier series of f(x) in $c \le x \le c + 2l$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Where a_0, an &b_n are called Fourier coefficients(or) Euler constants

4. Find the coefficient of b_n of cos 5x in the Fourier cosine series of the function

$f(x) = \sin 5x$ in the in the interval $(0, \pi)$.

Solution: The Fourier Cosine series is

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx = \frac{2}{\pi}\int_0^{\pi} \cos 5x \cos nx dx$$

$$= \frac{2}{\pi}\int_0^{\pi} [\cos(5+n)x + \cos(5-n)x] d$$

$$= \frac{2}{\pi}\left[\frac{\sin(5+n)x}{5+n} + \frac{\sin(5-n)x}{5-n}\right]_0^{\pi} = 0; \text{Therefore}, b_n = 0$$

5. Find the constant a_0 of the Fourier series for the function of $f(x) = x$ in $0 \le x \le 2\pi$

Solution:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\int_0^{2\pi} x dx = \frac{1}{\pi}\left(\frac{x^2}{2}\right)_0^{2\pi} = 2\pi$$

6. Obtain the first term of the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi} = \frac{2}{3}\pi^2$$

7. If $f(x) = x^3, -\pi < x < \pi$. *Find the constant term of its Fourier series.*

Solution:

$f(x) = x^3$, is an odd function. Therefore, the fourier constants $a_0 = 0$

8. Find the constant term in the Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$

Solution:

$a_0 = 0$ since $f(x)$ is an odd function $(-\pi, \pi)$

9. Find the sum of the Fourier series of $f(x) = x + x^2$ in $-\pi < x < \pi$ at $x = \pi$.

Solution:

Given $f(x) = x + x^2$

The sum of Fourier series is equal to the arithmetic mean of the value of f(x)at $x = \pi$ and $x = -\pi$.

Sum of Fourier series$= \dfrac{f(\pi)+f(-\pi)}{2} = \dfrac{\pi+\pi^2-\pi+\pi^2}{2} = \pi^2$

10. What is the constant term a_0 and the coefficient of cosnx, a_n in the Fourier series of $f(x) = x - x^3$ in $(-\pi, \pi)$.

Solution:

$$f(x) = x - x^3 \Rightarrow f(-x) = -x + x^3$$

$$= -(x - x^3) = -f(x)$$

Therefore, f(x) is an odd function of x in $(-\pi, \pi)$. Therefore, the Fourier series of f(x) Contains sine terms only. Therefore, $a_0 = 0$ and $a_n = 0$

11.Find the root mean square value of the function f(x)= x^2 in the interval (0,1).

Solution:

$$\text{RMS value} = \sqrt{\frac{1}{1}\int_0^1 x^2\,dx} = \sqrt{\frac{1}{1}\left(\frac{x^3}{3}\right)_0^1} = \sqrt{\frac{1}{1}\left[\frac{1^3}{3}\right]} = \frac{1}{\sqrt{3}}$$

22. Define root mean square value of a function f(x) in a < x < b.

Solution:

$$\text{R.M.S.value } \bar{y} = \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx}$$

23 .State Parseval`s identity for full range expansion of f(x) as Fourier series in (0,2l).

Solution:

$$\frac{1}{l}\int_0^{2l} (f(x))^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) \text{ where } a_0, a_n \text{ and } b_n \text{ are}$$

Fourier coefficients in the expansion of $f(x)$ as a Fourier series .

25. What do you mean by Harmonic Analysis?

Solution:

The process of finding the Fourier series for a function given by numerical value is known as harmonic analysis. In harmonic analysis

$$a_0 = 2 \text{ (mean value of y in } (0,2\pi))$$

$$a_n = 2 \text{ (mean value of y Cos nx in } (0,2\pi))$$

$$b_n = 2\text{(mean value of y Sin nx in } (0,2\pi))$$