2M ⇒ Properties of eigen values and eigen vectors :-

⇒ seven properties :-

1, If λ is the eigen value of A, then $\frac{|A|}{\lambda}$ is the eigen value of adjoint (adj A).

2, If A square matrix (A) and its transpose (AT) have the same eigen values.

3, The eigen values of a real symmetric matrix are real.

4) sum of the eigen values equal to(=) sum of the main diagonal elements.

5) product of the eigen values = |A|

6) The eigen values of a triangular matrix are the diagonal elements.

7). If λ is the eigen value A, then $\frac{1}{\lambda}$ $(\lambda \neq 0)$ is the eigen value of A^{-1}.

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Problems :-

1) Find the sum and products of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$

Soln :- properties = 4

Sum of eigen values = sum of the main diagonal element

$= 1 + 0 + 3 = 4$

Properties = 5

Product of the eigen values = |A|

$$= 1\begin{vmatrix} 0 & 3 \\ -1 & 8 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ -2 & 3 \end{vmatrix} + 3\begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}$$

$$= 1(0+3) - 2(3+6) + 3(-1-0)$$

$$= 1(3) - 2(9) + 3(-1)$$

$$= 3 - 18 - 3$$

$$= -15 - 3$$

$$= -18$$

2, If 3, 6 are the eigen values of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, find the 3rd eigen values and A^{-1}.

Soln:- Let λ be 3rd eigen value.

Sum of the eigen values = sum of the main diagonal elements.

$$= 1+5+1 = 7$$

$$3+6+\lambda = 7$$

$$9+\lambda = 7$$

$$\lambda = 7-9 = -2$$

$$\therefore 3^{rd} \Rightarrow \lambda = -2$$

$\therefore$ The 3rd eign value is -2.

The eigen values are $-2, 3, 6$

The eigen values of A^{-1} are $\frac{1}{-2}, \frac{1}{3}, \frac{1}{6}$

3, If 3, 15 are eigen values, then find $|A|$ without expanding the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

Let λ be the 3rd eigen values.

Product of the eigen values $= |A|$.

$= 3 \times 15 \times 0 = |A|$

$= 0 = |A|$

$\Rightarrow |A| = 0.$

(4) Find $|A|$ of 3×3 [If sum of two eigen values is equal to trace of matrix]

Soln:- let λ_1, λ_2 and λ_3 be the eigen values.

Sum of two eigen values = trace of matrix

$$\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$$

$$\lambda_3 = 0$$

Now product of the eigen values $= |A|$

$$\lambda_1 \times \lambda_2 \times 0 = |A|$$

$$\Rightarrow |A| = 0$$

(5) when $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$, find the eigen values of A, A^{-1}, A^2, $5A^2$, adj A, $A^2 - 2I$

$3 - 2 = a + b.$

$1 = a+b$

$a+b = 1 \rightarrow (1)$

Product of the eigen values $= |A|$

$3 \times (-2) = |A|$

$-6 = ab - 4$

$ab = -6 + 4.$

$ab = -2$

The eigen values of $5A^2$ are 45, 20, 125

The eigen values of $A^2 - 2I$ are 7, 2, 23

Eigen values of adj A are = 10, 15, 6.

adj $A = |A| / \lambda$

By properties, product the eigen value $= |A|$

$= 3 \times 2 \times 5 = |A|$

$= |A| = 30$

The eigen values of adj A are $\frac{30}{2}, \frac{30}{3}, \frac{30}{5}$

$= 15, 10, 6$

(6) Find the constant a and b such that $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$. has 3 and -2 as its eigen values

$a = -\frac{2}{b}$

sub $a = -\frac{2}{b}$ in (1)

$-\frac{2}{b} + b = 1$

$\frac{-2 + b^2}{b} = 1$

$-2 + b^2 = b$

$b^2 - b - 2 = 0$

$(b-2)(b+1) = 0$

$b - 2 = 0$ | $b + 1 = 0$

$b = 2$ | $b = -1$

$\boxed{b = -1, 2}$

sub $b = -1$ in (1)

$a - 1 = 1$

$a = 2$

$\boxed{a = -1}$

sub $b = 2$ in (1)

$a + 2 = 1$

$a = 1 - 2$

$\boxed{a = -1}$

Properties :- 6

The eigen values of a triangular matrix are the diagonal elements.

$\therefore$ The eigen values of A are 3, 2, 5.

The eigen values of A^{-1} are $\frac{1}{3}, \frac{1}{2}, \frac{1}{5}$

The eigen value of A^2 are 9, 4, 25

Here, D_1 = sum of the main diagonal elements.

$= 1 + 2 = 3$

$D_2 = |A| = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$\therefore \lambda^2 - 3\lambda + 2 = 0$

ii) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$D_1 = 1 + 3 = 4$

$D_2 = |A| = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} = 3 - 8 = -5$

$\therefore \lambda^2 - 4\lambda - 5 = 0$

iii) $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

Let $A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

The characteristic equation is

$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

Here,

D_1 = sum of the main diagonal element.

$= 2 + 1 - 4 = -1$

D_2 = Sum of the minors of the main diagonal elements.

$D_2 = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$

$= (-4 - 6) + (-8 + 5) + (2 + 9)$

$= -10 - 3 + 11$

$D_2 = -2$

$$D_3 = |A| = \begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 3 \\ -5 & -4 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ -5 & 2 \end{vmatrix}$$

$$= 2(-4-6) + 3(-12+15) + 1(6+5)$$

$$= 2(-10) + 3(3) + 1(11)$$

$$= -20 + 9 + 11 = 0$$

$$\lambda^3 + \lambda^2 - 2\lambda - 0 = 0$$

iv)
$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

$D_1 = 8 + 7 + 3 = 18$

$$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$$= (21-16) + (24-4) + (56-36)$$

$$= 5 + 20 + 20 = 45$$

$$D_3 = 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} - (-6)\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$$

$$= 8(21-16) + 6(-18+8) + 2(24-14)$$

$$= 8(5) + 6(-10) + 2(10)$$

$$= 40 - 60 + 20 = 0$$

$$\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$$