$$A^{-1} = \frac{1}{5}\left[\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{5}\left[\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\right]$$

$$A^{-1} = \frac{1}{5}\left[\begin{pmatrix} 1-4 & 4-0 \\ 2-0 & 3-4 \end{pmatrix}\right]$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

17/10/23 4) Use cayley hamilton theorem, To find the value of $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = 0$ where Period:6

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

The characteristic equation

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$D_1 =$ sum of the main diagonal elements.

$$= 2 + 1 + 2$$

$$\boxed{D_1 = 5}$$

$D_2 =$ sum of the minors of main diagonal elements.

$$= (2-0) + (4-1) + (2-0)$$

$$= 2 + 3 + 2 = 7$$

$$D_3 = |A|$$

$$= 2(2-0) - 1(0) + 1(0-1)$$

$$= 2(2) + 1(-1)$$

$$= 4 - 1 = 3$$

$$\therefore \quad \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

using cayley hamilton theorem:-

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A + 8A^2 - 2A + I = 0$$

$$\begin{array}{r|l} & A^5 + A \\ \hline A^3 - 5A^2 + 7A - 3I & A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \\ & A^8 - 5A^7 + 7A^6 - 3A^5 \\ \hline & \qquad A^4 - 5A^3 + 8A^2 - 2A \\ & \qquad A^4 - 5A^3 + 7A^2 - 3A \\ \hline & \qquad\qquad A^2 + A + I \end{array}$$

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = (A^3 - 5A^2 + 7A - 3I)$$

$$(A^5 + A) + (A^2 + A + I)$$

$$= 0 + (A^2 + A + I)$$

$$= A^2 + A + I$$

$$A^2 = A \times A$$

$$= \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 0 \end{pmatrix}$$

$$A^2 + A + I = \begin{pmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{pmatrix}$$