



Centre of curvature & Circle of curvature
Centre of curvature at any Pt. on the curve

$y = f(x)$ at $C(\bar{x}, \bar{y})$

where $\bar{x} = x - \frac{dy}{dx} \left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \right]$

$$\bar{y} = y + \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$

(or)

$$\bar{x} = x - \frac{y_1 [1 + y_1^2]}{y_2}$$

$$\bar{y} = y + \left[\frac{1 + y_1^2}{y_2} \right]$$

Circle of curvature at any point is
 $(x - \bar{x})^2 + (y - \bar{y})^2 = r^2$ where r is the radius of curvature.

Q. Find the centre and circle of curvature at (c, c) on $xy = c^2$.

Soln.

Given $xy = c^2$

$$x \frac{dy}{dx} + y = 0$$

$$x \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

At (c, c) , $y_1 = -1$



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Unit 3-Differential Calculus

Centre of Curvature

$$\text{and } \frac{d^2 y}{dx^2} = \frac{-[x y_1 - y(1)]}{x^2} = \frac{-x y_1 + y}{x^2}$$

$$\text{At } (c, c), \quad y_2 = \frac{-c(-1) + c}{c^2} = \frac{2c}{c^2}$$

$$y_2 = \frac{2}{c}$$

$$\begin{aligned} \therefore \rho &= \frac{[1 + y_1^2]^{3/2}}{y_2} \\ &= \frac{[1 + (-1)^2]^{3/2}}{2/c} \\ &= \frac{2^{3/2}}{2/c} = \frac{c}{2} \cdot 2\sqrt{2} \\ &= \sqrt{2}c \end{aligned}$$

To find \bar{x} & \bar{y} :

$$\begin{aligned} \bar{x} &= x - \frac{y_1 [1 + y_1^2]}{y_2} = x + \frac{1[1 + (-1)^2]}{2/c} \\ &= x + 2 \times \frac{c}{2} \end{aligned}$$

$$\bar{x} = x + c$$

$$\bar{y} = y + \frac{[1 + y_1^2]}{y_2} = y + \frac{[1 + (-1)^2]}{2/c}$$

$$= y + 2 \times \frac{c}{2}$$

$$\bar{y} = y + c$$

$$\text{At } (c, c): \quad \bar{x} = c + c = 2c$$

$$\bar{y} = c + c = 2c$$

Centre of curvature $C(\bar{x}, \bar{y}) = C(2c, 2c)$

Circle of curvature $(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$

$$(x - 2c)^2 + (y - 2c)^2 = (c\sqrt{2})^2$$