7). Find the circle of curvature $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $(a/4, a/4)$

Soln.

we know that $y_1 = -1$

$$y_2 = \frac{4}{a}$$

$$\therefore \rho = \frac{a}{\sqrt{2}}$$

To find $\bar{x}$ & $\bar{y}$:

$$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2} = x + \frac{1(1+(-1)^2)}{4/a}$$

$$= x + 2 \times \frac{a}{4}$$

$$\bar{x} = x + \frac{a}{2}$$

$$\bar{y} = y + \frac{[1+y_1^2]}{y_2} = y + \frac{[1+1]}{4/a}$$

$$= y + \frac{a}{2}$$

At $(a/4, a/4)$, $\bar{x} = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$

$$\bar{y} = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$$

centre of curvature $C(\bar{x}, \bar{y}) = C\left(\frac{3a}{4}, \frac{3a}{4}\right)$

circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$$\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$$

8). Find the circle of curvature $y^2 = 12x$ at $(3,6)$

Soln. :

Givn. $y^2 = 12x$

$$2y\frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = \frac{12}{2y} = \frac{6}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-6}{y^2}$$

At $(3, 6)$, $y_1 = 1$, $y_2 = -\frac{1}{6}$

$$\therefore \rho = \frac{[1+1]^{3/2}}{-1/6} = -6(2^{3/2}) = 2\sqrt{2}(-6)$$

$$= -12\sqrt{2}$$

$$\rho = 12\sqrt{2}$$

$$\bar{x} = x - \frac{y_1[1+y_1^2]}{y_2} = x - \frac{1[1+1]}{-1/6}$$

$$= x + 6(2)$$

$$= x + 12$$

$$\bar{y} = y + \frac{[1+y_1^2]}{y_2} = y + \frac{[1+1]}{-1/6} = y - 12$$

At $(3, 6)$, $\bar{x} = 3 + 12 = 15$

$\bar{y} = 6 - 12 = -6$

centre of curvature $C(\bar{x}, \bar{y}) = C(15, -6)$

circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

ie., $(x-15)^2 + (y+6)^2 = [12\sqrt{2}]^2$

Hw

Radius

$xy = 12$ at $(3, 4)$

$x^4 + y^4 = 2$ at $(1, 1)$

$\sqrt{x} + \sqrt{y} = 1$ at $(1/4, 1/4)$

centre & circle of curvature

$xy = 12$

$y = x^3$ at $(3, 27)$

4. Find the circle of curvature for $x^3+y^3=3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

Soln.

WKT $\frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = -\frac{32}{3a}$$

$$\rho = \frac{[1+(-1)^2]^{3/2}}{-\frac{32}{3a}}$$

$$= -2^{3/2}\frac{3a}{32} = -2\sqrt{2}\frac{3a}{32}$$

$$= -\frac{3\sqrt{2}a}{16}$$

$$\therefore \rho = \frac{3\sqrt{2}a}{16}$$

$$\bar{x} = x - \frac{y_1[1+y_1^2]}{y_2} = x + \frac{1(1+(-1)^2)}{-32/3a}$$

$$= x - \frac{3a}{32}(2)$$

$$\bar{x} = x - \frac{3a}{16}$$

$$\bar{y} = y + \frac{[1+y_1^2]}{y_2} = y + \frac{[1+(-1)^2]}{-32/3a}$$

$$= y - \frac{3a}{32}(2)$$

$$\bar{y} = y - \frac{3a}{16}$$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right)$, $\bar{x} = \frac{3a}{2} - \frac{3a}{16} = \frac{24a-3a}{16}$

$$\bar{x} = \frac{21a}{16}$$

and $\bar{y} = \frac{3a}{2} - \frac{3a}{16} = \frac{21a}{16}$

centre of curvature $C(\bar{x}, \bar{y}) = C\left(\frac{21a}{16}, \frac{21a}{16}\right)$

circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$$\left(x - \frac{21a}{16}\right)^2 + \left(y - \frac{21a}{16}\right)^2 = \left(\frac{3\sqrt{2}a}{16}\right)^2$$