



Unit 3-Differential Calculus

Circle of Curvature

Q). Find the circle of curvature $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(\frac{a}{4}, \frac{a}{4})$

Soln.

We know that $y_1 = -1$

$$y_2 = \frac{4}{a}$$

$$\therefore r = \frac{a}{\sqrt{2}}$$

To find \bar{x} & \bar{y} :

$$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2} = x + \frac{1(1+(-1)^2)}{4/a}$$

$$= x + 2 \times \frac{a}{4}$$

$$\bar{x} = x + \frac{a}{2}$$

$$\bar{y} = y + \frac{1+y_1^2}{y_2} = y + \frac{1+1}{4/a}$$

$$= y + \frac{a}{2}$$

$$At (\frac{a}{4}, \frac{a}{4}), \quad \bar{x} = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$$

$$\bar{y} = \frac{a}{4} + \frac{a}{2} = \frac{3a}{4}$$

Centre of curvature $C(\bar{x}, \bar{y}) = C(\frac{3a}{4}, \frac{3a}{4})$

Circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = r^2$

$$(x - \frac{3a}{4})^2 + (y - \frac{3a}{4})^2 = (\frac{a}{\sqrt{2}})^2$$

Q). Find the circle of curvature $y^2 = 12x$ at $(3, 6)$

Soln.:

Given $y^2 = 12x$

$$2y \frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = \frac{12}{2y} = \frac{6}{y}$$



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$$B \text{ for } \frac{d^2y}{dx^2} = -\frac{6}{y^2} \text{ equations for circle get from B.}$$

$$\text{At } (3, 6), \quad y_1 = 1, \quad y_2 = -\frac{1}{6}$$

$$\therefore r = \frac{\sqrt{1+1}}{-\frac{1}{6}} = -6(2^{3/2}) = 2\sqrt{2}(-6) \\ = -12\sqrt{2}$$

$$r = 12\sqrt{2}$$

$$\bar{x} = x - \frac{y_1[1+y_1^2]}{y_2} = x - \frac{1[1+1]}{-\frac{1}{6}} = x + 12$$

$$= x + 12$$

$$= x + 12$$

$$\bar{y} = y + \frac{1[1+y_1^2]}{y_2} = y + \frac{1[1+1]}{-\frac{1}{6}} = y - 12$$

$$\text{At } (3, 6), \quad \bar{x} = 3 + 12 = 15$$

$$\bar{y} = 6 - 12 = -6$$

centre of curvature $C(\bar{x}, \bar{y}) = C(15, -6)$

circle of curvature $(x - \bar{x})^2 + (y - \bar{y})^2 = r^2$
 $\text{ie., } (x - 15)^2 + (y + 6)^2 = [12\sqrt{2}]^2$

How $xy = 12$ at $(3, 4)$

$$x^4 + y^4 = 2 \text{ at } (1, 1)$$

$$\sqrt{x} + \sqrt{y} = 1 \text{ at } (\frac{1}{4}, \frac{1}{4})$$

centre & circle of curvature

$$xy = 12 \text{ at } (3, 2)$$

$$y = x^3 \text{ at } (3, 27)$$



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4. Find the circle of curvature for $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$.

Soln.

$$\text{WKT} \quad \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

$$\text{At } (\frac{3a}{2}, \frac{3a}{2}) \quad \frac{d^2y}{dx^2} = -1$$

$$\frac{d^2y}{dx^2} = -\frac{32}{3a}$$

$$r = \frac{[1 + (-1)^2]^{3/2}}{-\frac{32}{3a}}$$

$$= -2^{3/2} \cdot \frac{3a}{32} = -2\sqrt{2} \cdot \frac{3a}{32}$$

$$= -\frac{3\sqrt{2}a}{16}$$

$$\therefore r = \frac{3\sqrt{2}a}{16}$$

$$\bar{x} = x - \frac{y_1 [1 + y_1^2]}{y_2} = x + \frac{1(1 + (-1)^2)}{-32/3a}$$

$$= x - \frac{3a}{32} \quad (2)$$

$$\bar{x} = x - \frac{3a}{16}$$

$$\bar{y} = y + \frac{[1 + y_1^2]}{y_2} = y + \frac{[1 + (-1)^2]}{-32/3a}$$

$$= y - \frac{3a}{32} \quad (2)$$

$$\bar{y} = y - \frac{3a}{16}$$



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$$\text{At } \left(\frac{3a}{2}, \frac{3a}{2}\right), \quad \bar{x} = \frac{3a}{2} - \frac{3a}{16} = \frac{24a - 3a}{16}$$

$$\bar{x} = \frac{21a}{16}$$

$$\text{and } \bar{y} = \frac{3a}{2} - \frac{3a}{16} = \frac{21a}{16}$$

centre of curvature $C(\bar{x}, \bar{y}) = C\left(\frac{21a}{16}, \frac{21a}{16}\right)$

circle of curvature $(x - \bar{x})^2 + (y - \bar{y})^2 = r^2$

$$\left(x - \frac{21a}{16}\right)^2 + \left(y - \frac{21a}{16}\right)^2 = \left(\frac{3\sqrt{2}a}{16}\right)^2$$