SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
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COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT II
ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

(3) Write the quadratic form for the following matrix

(i) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$
(ii) $\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$

Soln :

(i) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$

$Q = a_{11} x_1^2 + a_{22} x_2^2 + a_{33} x_3^2 + 2 a_{12} x_1 x_2 + 2 a_{23} x_2 x_3 + 2 a_{31} x_3 x_1$

$Q = x_1^2 + 2 x_2^2 + 3 x_3^2 + 2 x_1 x_2 + 2 x_2 x_3 - 2 x_3 x_1$

(ii) $Q = 2 x_1^2 + x_2^2 - 3 x_3^2 + 12 x_1 x_2 - 8 x_2 x_3 - 4 x_1 x_3 .$

Canonical form :

For a real quadratic form $Q = x^T A x$, the canonical form is $y^T D Y$ (or)

$\lambda_1 y_1^2 + \lambda_2 y_2^2 + \cdots + \lambda_n y_n^2$
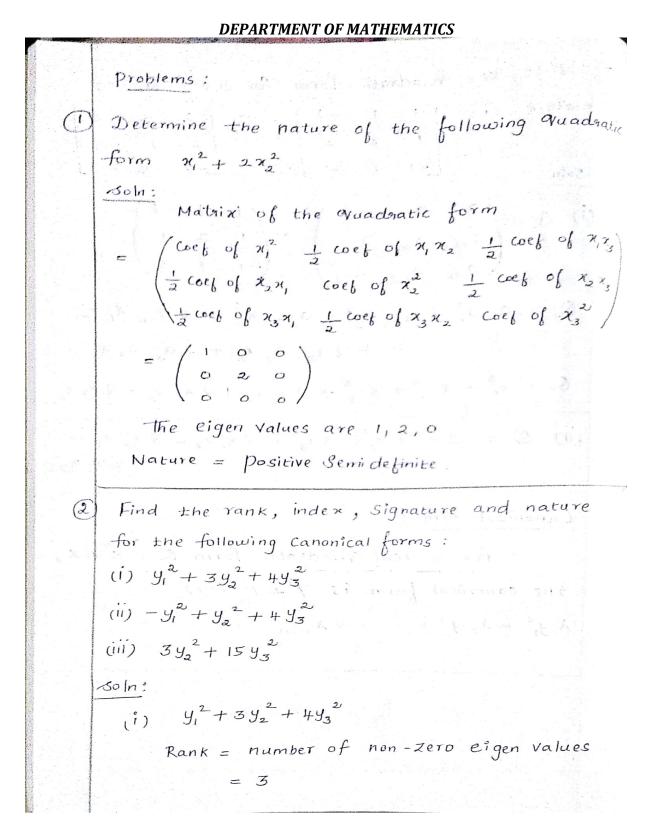
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Problems :

① Determine the nature of the following quadratic form $x_1^2 + 2x_2^2$

Soln :

Matrix of the quadratic form

$$= \begin{pmatrix} \text{Coef of } x_1^2 & \frac{1}{2}\text{coef of } x_1 x_2 & \frac{1}{2}\text{coef of } x_1 x_3 \\ \frac{1}{2}\text{coef of } x_2 x_1 & \text{coef of } x_2^2 & \frac{1}{2}\text{coef of } x_2 x_3 \\ \frac{1}{2}\text{coef of } x_3 x_1 & \frac{1}{2}\text{coef of } x_3 x_2 & \text{coef of } x_3^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The eigen values are 1, 2, 0

Nature = Positive Semi definite

② Find the rank, index, Signature and nature for the following Canonical forms :

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

(iii) $3y_2^2 + 15y_3^2$

Soln:

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

Rank = number of non-zero eigen values

= 3
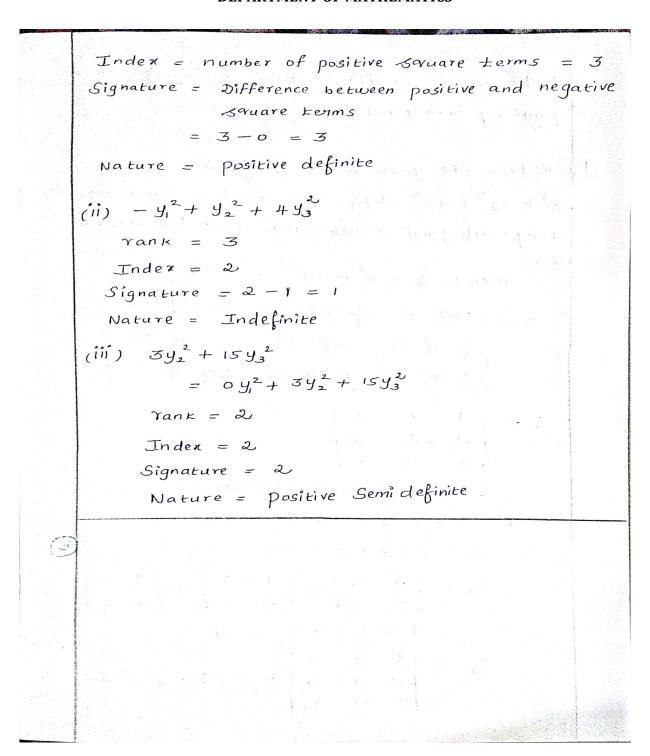
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Index = number of positive square terms = 3

Signature = Difference between positive and negative square terms

$$= 3 - 0 = 3$$

Nature = positive definite

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

rank = 3

Index = 2

Signature = $2 - 1 = 1$

Nature = Indefinite

(iii) $3y_2^2 + 15y_3^2$

$$= 0y_1^2 + 3y_2^2 + 15y_3^2$$

rank = 2

Index = 2

Signature = 2

Nature = positive Semi definite

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① $\quad 2x^2 + 3y^2 + 2z^2 + 2xy$

$$= \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{ccc} x & y & z \\ \left[\begin{array}{ccc} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{array}\right] \end{array}$$

② $\quad 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{ccc} x_1 & x_2 & x_3 \\ \left[\begin{array}{ccc} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{array}\right] \end{array}$$

$$\begin{array}{|ccc|} \hline 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \\ \hline \end{array}$$