UNIT I
MATRIX EIGEN VALUE PROBLEMS

Characteristic Equations :

Let A be a given matrix.

Let λ be a Scalar.

The equation $|A - \lambda I| = 0$ is called the characteristic equation of the matrix A.

Note :

★ For any square matrix A, the sum of the eigen values of a matrix is equal to trace of the matrix.

★ For a 2×2 matrix, the characteristic equation is,

$$\lambda^2 - C_1 \lambda + C_2 = 0$$

where C_1 = Sum of the main diagonal elements

$C_2 = |A|$

★ For a 3×3 matrix, the characteristic equation is,

$$\lambda^3 - C_1 \lambda^2 + C_2 \lambda - C_3 = 0$$

where C_1 = Sum of the main diagonal elements

C_2 = Sum of the minors of the main diagonal elements.

$C_3 = |A|$.

PROBLEMS :

(1) Find the characteristic equation of $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

Solution : Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

The characteristic equation is,

$$\lambda^2 - c_1 \lambda + c_2 = 0 \longrightarrow (1)$$

$$c_1 = 1 + 2 = 3$$

$$c_2 = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2$$

subs c_1 & c_2 in (1),

$$\boxed{\lambda^2 - 3\lambda + 2 = 0}$$

(2) Find the characteristic equation of $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

Solution : Let $A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

The characteristic equation is,

$$\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0 \longrightarrow (1)$$

$$c_1 = 2 + 1 - 4 = -1$$

$$c_2 = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= -10 - 3 + 11 = -2$$

$$c_3 = \begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{vmatrix} = 0 .$$

Subs c_1, c_2 & c_3 in (1),

$$\boxed{\lambda^3 + \lambda^2 - 2\lambda = 0}$$

Problems :

Find the characteristic polynomial of

(1) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ Soln : $\lambda^2 - 4\lambda - 5$

(2) $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ Soln : $\lambda^2 - 5\lambda + 7$

(3) $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ Soln : $\lambda^3 - 18\lambda^2 + 45\lambda = 0$

(4) $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ Soln : $\lambda^3 - 3\lambda^2 + 2\lambda = 0$

(5) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ Soln : $\lambda^3 - 7\lambda^2 + 36 = 0$