



DEPARTMENT OF MATHEMATICS

Type II : Symmetric matrix with repeated roots :

① Find the eigen values and eigen vectors of the

matrix
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Soln :

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

The char eqn is,

$$\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0$$

$$c_1 = 0$$

$$c_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1 - 1 - 1 = -3$$

$$c_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2$$

$$\lambda^3 - 3\lambda - 2 = 0 \rightarrow \textcircled{1}$$

$$\lambda = -1, -1, 2$$

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \textcircled{2}$$



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Case (i) : $\lambda = 2$

$$\textcircled{2} \Rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{3} = \frac{x_2}{3} = \frac{x_3}{3}$$

\therefore The eigen vector is $x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Case (ii) : $\lambda = -1$

$$\textcircled{2} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

All the three rows are equal.

Taking the first row,

$$x_1 + x_2 + x_3 = 0$$

Put $x_1 = 0$,

$$x_2 + x_3 = 0$$

$$\Rightarrow x_2 = -x_3$$

$$\Rightarrow \frac{x_2}{1} = \frac{x_3}{-1}$$

$$\therefore x_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$



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The given matrix is a symmetric matrix.

In this matrix, x_3 is orthogonal to x_1 and x_2 .

$$\text{Let } x_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$x_3 \text{ is orthogonal to } x_1 \Rightarrow x_3^T x_1 = 0$$

$$(a \ b \ c) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$a + b + c = 0 \rightarrow (i)$$

$$x_3 \text{ is orthogonal to } x_2 \Rightarrow x_3^T x_2 = 0$$

$$(a \ b \ c) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$$

$$0a + b - c = 0 \rightarrow (ii)$$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & +1 \\ 1 & -1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$\therefore x_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

Eigen values : λ : -1 -1 2

Eigen vectors : x : $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$



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② Find the eigen values and eigen vectors of

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Soln: $\lambda = 14, 0, 0$

$$X = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} 13 \\ -2 \\ -3 \end{pmatrix}$$

③ $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

Soln: $\lambda = 1, 3, 3$

$$X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Type 3: Non-Symmetric matrix with repeated roots:

① Find all the eigen values and eigen vectors of the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

Soln:

$$\text{Let } A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$



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$$C_1 = -1$$

$$C_2 = \begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$$
$$= -21$$

$$C_3 = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix} = 45$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0.$$

$$\lambda = -3, -3, 5$$

$$(A - \lambda I)x = 0.$$

$$\begin{pmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Case (i): $\lambda = 5$

$$\begin{pmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -3 & -7 \\ -6 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-24} = \frac{x_2}{-48} = \frac{x_3}{24}$$

$$x_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$



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Case (ii) : $\lambda = -3$

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

All the three equations are same.

$$x_1 + 2x_2 - 3x_3 = 0$$

Put $x_1 = 0$,

$$2x_2 = 3x_3$$

$$\frac{x_2}{3} = \frac{x_3}{2}$$

$$\therefore x_2 = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

Put $x_2 = 0$,

$$x_1 = 3x_3$$

$$\frac{x_1}{3} = \frac{x_3}{1}$$

$$\therefore x_3 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

Eigen values : 5 -3 -3

Eigen vectors : $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

② $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$ Soln : λ : -2 2 2
 x : $\begin{pmatrix} 4 \\ 1 \\ -7 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

③ $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ Soln : λ : 1 1 5
 x : $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$