

### SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution)

#### **DEPARTMENT OF MATHEMATICS**

# UNIT II ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX Problems:

1) Reduce the Gruaduatic form 6x2+3y2+3z2-4xy-242+42x into Canonical form by an orthogonal transformation. Discuss its nature. Solution:  $Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \end{bmatrix}$  $\lambda^3 - c_1 \lambda^2 + c_2 \lambda - c_3 = 0 \rightarrow 0$ C. = 6 + 3 + 3 = 12  $c_2 = \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 3 & 3 \end{vmatrix} = 36$  $C_3 = \begin{vmatrix} b & -2 & 2 \\ -2 & 3 & -1 \end{vmatrix} = 32$  $\lambda^{3} - 12 \lambda^{2} + 36 \lambda - 32 = 0$ λ = 2,2,8  $\begin{pmatrix} 6 - \lambda & -2 & 2 \\ -2 & 3 - \lambda & -1 \\ & & & \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ & & \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ 



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$$\frac{Case(i): A = S}{\begin{pmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

$$\frac{\chi_1}{12} = \frac{\chi_2}{-6} = \frac{\chi_3}{6}$$

$$\chi_1 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

Case (ii) : 1 = 2

$$\begin{pmatrix} A_1 & -A_2 & A_2 \\ -A_2 & 1 & -I \\ A_2 & -I & I \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} O \\ O \\ O \end{pmatrix}$$

All the three rows are equal

Put 
$$x_1 = 0$$
,  $-2x_2 = -2x_3$ 

$$\frac{x_2}{1} = \frac{x_3}{1}$$

$$\therefore x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The given matrix is a symmetric matrix. In this matrix, the third eigen value vector  $X_3$  is orthogonal to  $x_1 \& x_2$ .  $X_3^T X_1 = 0 \implies (a \ b \ c) \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 0 \implies (a - b + c = 0 \implies (i)$   $X_3^T X_2 = 0 \implies (a \ b \ c) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 \implies 0 \ a + b + c = 0 \implies (ii)$ 



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Solving (i) & (ii),
$$X_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

The modal matrix is .

$$M = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

The normalized madrix is,

$$N = \begin{pmatrix} 2/\sqrt{6} & 0 & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}$$

$$N^TAN = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \mathcal{D}$$

Canonical form :

$$y^{T} \mathcal{D} Y = (y_{1} \quad y_{2} \quad y_{3}) \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \\ y_{3} \end{pmatrix}$$

$$= 8y_{1}^{2} + 2y_{2}^{2} + 2y_{3}^{2} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_{1} \\ y_{2} \\ y_{3} \end{pmatrix}$$

Index = No. of positive square terms = 3

Rank = No. of non-zero eigen values = 3

Signature = 25-r = 6-3 = 3

The Quadratic form is positive definite.