SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Example 5.34 Design a Chebyshev filter for the following specification using (a) bilinear transformation (b) impulse invariance Method.

$$0.8 \leq |H(e^{j\omega})| \leq 1 \qquad 0 \leq \omega \leq 0.2\pi$$
$$|H(e^{j\omega})| \leq 0.2 \qquad 0.6\pi \leq \omega \leq \pi$$

Solution

(a) Given $\omega_s = 0.6\pi, \omega_p = 0.2\pi$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2 \Rightarrow \lambda = 4.899$$
$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \Rightarrow \varepsilon = 0.75$$
$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 0.6498 \quad (\because\ T = 1 \text{ sec})$$
$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2.752$$
$$N = \frac{\cosh^{-1}\lambda/\varepsilon}{\cosh^{-1}1/k} = 1.208$$
$$\Rightarrow N = 2$$

$$\Rightarrow N = 2$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 3$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right] = 0.3752$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right] = 0.75$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2$$

$$s_k = a \cos \phi_k + jb \sin \phi_k$$

$$s_1 = -0.2653 + j0.53$$

$$s_2 = -0.2653 - j0.53$$

Denominator of

$$H(s) = (s + 0.2653)^2 + (0.53)^2$$
$$= s^2 + 0.5306s + 0.3516$$

For N even, Numerator of $H(s)$ is $\dfrac{0.3516}{[1 + (0.75)^2]^{1/2}} = 0.28$

$$H(s) = \frac{0.28}{s^2 + 0.5306s + 0.3516}$$

Using bilinear transformation

$$H(z) = H(z)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} \qquad \boxed{\because T = 1 \text{ sec}}$$

$$H(z) = \frac{0.28(1 + z^{-1})^2}{5.4128 - 7.298z^{-1} + 3.29z^{-2}}$$
$$= \frac{0.052(1 + z^{-1})^2}{1 - 1.3480z^{-1} + 0.608z^{-2}}$$

(b) By using impulse invariance method

$$\omega = \Omega T \Rightarrow \omega_p = \Omega_p T \quad \text{and} \quad \omega_s = \Omega_s T$$

For $T = 1$ sec

$$\frac{\omega_s}{\omega_p} = \frac{\Omega_s}{\Omega_p} = \frac{0.6\pi}{0.2\pi} = 3$$

$$N = \frac{\cosh^{-1}\frac{\lambda}{\varepsilon}}{\cosh^{-1}\frac{1}{k}} = \frac{\cosh^{-1}\frac{4.899}{0.75}}{\cosh^{-1} 3} = 1.45$$

Approximating N to next higher integer, we get $N = 2$. We know $\mu = 3$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.3627$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.7255$$

$$\phi_1 = 135^\circ; \phi_2 = 225^\circ$$

$$s_1 = -0.2564 + j0.513$$

$$s_2 = -0.2564 - j0.513$$

Numerator of $H(s) = 0.264$

$$H(s) = \frac{0.264}{s^2 + 0.513s + 0.33} = \frac{(0.5146)(0.513)}{(s + 0.2564)^2 + (0.513)^2}$$

Taking inverse Laplace transform we obtain

$$h(t) = 0.5146e^{-0.2564t}\sin 0.513t$$

Let $t = nT$. Then $h(nT) = 0.5146e^{-0.2564nT}\sin 0.513nT$.

The z-Transform

$$H(z) = \frac{0.5146e^{-0.2564T}z^{-1}\sin 0.513T}{1 - 2e^{-0.2564T}z^{-1}\cos 0.513T + e^{-0.513T}z^{-2}}$$

Assume $T = 1$ sec

$$H(z) = \frac{0.1954z^{-1}}{1 - 1.3483z^{-1} + 0.5987z^{-2}}$$

(or)

$$H(s) = \frac{0.264}{s^2 + 0.513s + 0.33} = \frac{0.257j}{s + 0.256 - j0.513} - \frac{0.257j}{s + 0.256 + j0.513}$$

$$H(z) = \frac{0.257j}{1 - e^{-0.256T}e^{j0.513T}z^{-1}} - \frac{0.257j}{1 - e^{-0.256T}e^{-j0.513T}z^{-1}}$$

$$= \frac{0.1954z^{-1}}{1 - 1.3483z^{-1} + 0.5987z^{-2}}$$

Example 5.39 Design a digital Chebyshev filter to meet the constraints

$$\frac{1}{\sqrt{2}} \leq H(e^{j\omega}) \leq 1 \qquad \text{for } 0 \leq \omega \leq 0.2\pi$$

$$0 \leq |H(e^{j\omega})| \leq 0.1 \qquad \text{for } 0.5\pi \leq \omega \leq \pi$$

by using bilinear transformation and assume sampling period $T = 1$ sec.

(AU ECE May'05)

Solution Given $\omega_s = 0.5\pi; \omega_p = 0.2\pi$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.1 \Rightarrow \lambda = 9.95$$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}} \Rightarrow \varepsilon = 1$$

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan(0.1\pi) = 0.65$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2\tan\left(\frac{\pi}{4}\right) = 2$$

$$N = \frac{\cos h^{-1}\frac{\lambda}{\varepsilon}}{\cos h^{-1}\frac{\Omega_s}{\Omega_p}} = \frac{\cos h^{-1} 9.95}{\cos h^{-1}\left(\frac{2}{0.65}\right)} = 1.669$$

approximate $N = 2$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 1 + \sqrt{2} = 2.414$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.65\left[\frac{2.414^{1/2} - 2.414^{-1/2}}{2}\right] = 0.295$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.65\left[\frac{2.414^{1/2} + 2.414^{-1/2}}{2}\right] = 0.717$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2; \quad \phi_1 = 135^\circ; \ \phi_2 = 225^\circ$$

$$s_k = a\ \cos\phi_k + j\ b\ \sin\phi_k$$

$$s_1 = 0.295\cos 135^\circ + j\,0.717\sin 135^\circ = -0.2086 + j0.507$$

$$s_2 = 0.295\cos 225^\circ + j\,0.717\sin 225^\circ = -0.2086 - j0.507$$

Denominator of

$$H(s) = (s + 0.2086)^2 + (0.507)^2 = s^2 + 0.4172s + 0.3$$

For N even, numerator of $H(s)$ is

$$= \frac{0.3}{\sqrt{1+\varepsilon^2}} = 0.212$$

$$H(s) = \frac{0.212}{s^2 + 0.4172s + 0.3}$$

Using bilinear transformation

$$H(z) = H(s)|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Since $T = 1$

$$\begin{aligned} H(z) &= \frac{0.212(1+z^{-1})^2}{4(1-z^{-1})^2 + 0.8344(1-z^{-2}) + 0.3(1+z^{-1})^2} \\ &= \frac{0.212(1+2z^{-1}+z^{-2})}{4 - 8z^{-1} + 4z^{-2} + 0.8344 - 0.8344z^{-2} + 0.3 + 0.6z^{-1} + 0.3z^{-2}} \\ &= \frac{0.212(1+2z^{-1}+z^{-2})}{5.1344 - 7.40z^{-1} + 3.4656z^{-2}} \\ &= \frac{0.0413(1+z^{-1})^2}{1 - 1.44z^{-1} + 0.675z^{-2}} \end{aligned}$$

Thank You!