



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS 23MAT101 - MATRICES AND CALCULUS UNIT-I MATRIX EIGENVALUE PROBLEM

Eigen Values & Eigen Vectors of a real matrix

Let $A = [a_{ij}]$ be a square matrix.

The Char eqn of A is $|A - \lambda I| = 0$

The roots of the Char eqn are:

Called eigen values of A .

If there exists a non-zero vector

$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, such that $AX = \lambda X$, then the

vector X is called an eigen vector of A
Corresponding to the eigen value of A .

Problems

1) Find the eigen values and eigen vectors
of the matrix $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$.

Soln:

Step 1: To find the Char eqn $|A - \lambda I| = 0$

$$(6) \quad \lambda^2 - c_1\lambda + c_2 = 0 \rightarrow \textcircled{1}$$

$$c_1 = 1 - 1 = 0$$

$$c_2 = |A| = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1 - 3 = -4$$

$$\textcircled{1} \Rightarrow \lambda^2 - 0\lambda - 4 = 0 \Rightarrow \lambda^2 - 4 = 0$$

Step 2: To find eigen values

$$\lambda^2 - 4 = 0$$

$$\lambda^2 = 4$$

$$\lambda = \pm 2$$

$$\boxed{\lambda = 2, -2}$$



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step 3: To find eigen vectors

$$(A - dI)x = 0$$

$$\left[\begin{pmatrix} 1 & -1 \\ 3 & -1 \end{pmatrix} - d \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & -1 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-d & -1 \\ 3 & -1-d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \textcircled{2}$$

Case (i) $d = 2$

$$\textcircled{2} \Rightarrow \begin{pmatrix} 1-2 & -1 \\ 3 & -1-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-x_1 - x_2 = 0 \Rightarrow x_1 = -x_2$$

$$3x_1 - 3x_2 = 0 \Rightarrow 3x_1 = 3x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{1}$$

$$\therefore x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Case (ii) $d = -2$

$$\textcircled{2} \Rightarrow \begin{pmatrix} 1+2 & -1 \\ 3 & -1+2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$3x_1 - x_2 = 0$$

$$3x_1 + x_2 = 0$$

Both are same. \therefore we can take only one equation,



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$$3x_1 + x_2 = 0$$

$$3x_1 = -x_2$$

$$\frac{x_1}{-1} = \frac{x_2}{3}$$

$$\therefore X_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

Eigen values	2	-2
Eigen Vectors	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$