Eigen Values & Eigen Vectors of a real matrix

Let $A = [a_{ij}]$ be a square matrix.
The Char equ of A is $|A - \lambda I| = 0$
The roots of the char equ are called eigen values of A.

If there exists a non-zero vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, such that $AX = \lambda X$, then the vector X is called an eigen vector of A corresponding to the eigen value of A.

Problems

1) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$.

Soln:

Step 1: To find the char. equ $|A - \lambda I| = 0$

(ie) $\lambda^2 - c_1\lambda + c_2 = 0 \longrightarrow (1)$

$c_1 = 1 - 1 = 0$

$c_2 = |A| = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1 - 3 = -4$

$(1) \Rightarrow \lambda^2 - 0\lambda - 4 = 0 \Rightarrow \lambda^2 - 4 = 0.$

Step 2: To find eigen Values

$\lambda^2 - 4 = 0$

$\lambda^2 = 4$

$\lambda = \pm 2$

$\boxed{\lambda = 2, -2}$

Step 3: To find eigen vectors

$$(A-\lambda I)X = 0$$

$$\left[\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow (2)$$

Case (i) $\lambda = 2$

$$(2) \Rightarrow \begin{pmatrix} 1-2 & 1 \\ 3 & -1-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-x_1 + x_2 = 0 \Rightarrow x_1 = x_2$$

$$3x_1 - 3x_2 = 0 \Rightarrow 3x_1 = 3x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow \frac{x_1}{1} = \frac{x_2}{1}$$

$$\therefore X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Case (ii) $\lambda = -2$

$$(2) \Rightarrow \begin{pmatrix} 1+2 & 1 \\ 3 & -1+2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$3x_1 + x_2 = 0$$

$$3x_1 + x_2 = 0$$

Both are same. $\therefore$ we can take only one equation,

$3x_1 + x_2 = 0$

$3x_1 = -x_2$

$\therefore \frac{x_1}{-1} = \frac{x_2}{3}$

$\therefore X_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Eigen values	2	−2
Eigen Vectors	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$