CHARACTERISTIC EQUATION:

Let A be a given matrix

Let λ be a scalar

The equation $|A - \lambda I| = 0$ is called Characteristic equation of the matrix A.

1. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

Solution:

The characteristic equation is given by

$$\lambda^2 - S_1\lambda + S_2 = 0.$$

S_1 = Sum of the diagonal elements

$= 1 + 4 = 5$

$S_2 = |A|$

$= \begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} = 4-(-6) = 4+6 = 10$

$S_2 = 10$

$\therefore$ The Characteristic equation is

$\lambda^2 - 5\lambda + 10 = 0$

HW.

$A = \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix}$

S_1 (15) $S_2 = 56 - 8$

$S_2 = 48$

d) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 5 & 6 \\ 4 & 5 & 2 \end{bmatrix}$ Find the characteristic equation

The characteristic equation is given by

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

S_1 = Sum of the diagonal elements

$= 5+2+1 = 8$

S_2 = Sum of the minors of the diagonal elements

$= \begin{vmatrix} 5 & 6 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$

$= (10 - 30) + (2 - 12) + (5 - 2)$

$= -20 - 10 + 3$

$S_2 = -27$

$S_3 = |A|$

$= -20 - 1(4 - 24) + 3(10 - 20)$

$= -20 + 20 + 3(-10)$

$S_3 = -30$

The characteristic equation is

$\lambda^3 - 8\lambda^2 - 27\lambda + 30 = 0.$

3) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

The characteristic equation is given by

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$

S_1 = Sum of the diagonal elements

$= 3 + 3 + 3 = 9$

S_2 = Sum of the minors of the diagonal elements

$= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}.$

$= (9-1) + (3+1) + (-1+3)$

$= 8+8+8 = 24$

$S_3 = |A|$

$= 3(9-1) - 1(3+1) + 1(-1-3)$

$= 24 - 4 - 4$

$S_3 = 16$

∴ The characteristic equation is

$\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$

4) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

The characteristic equation is given by

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$

S_1 = Sum of the diagonal elements

$= 7+6+5$

$= 18$

S_2 = Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 6 & -2 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= (30-4) + (35-0) + (42-4)$$

$$= 26 + 35 + 38$$

$$= 99$$

$S_3 = |A|$

$$= 7(30-4) - (-2)(-10-0) + 0(4-0)$$

$$= 7(26) + 2(-10)$$

$$= 182 - 20 = 162$$

∴ The characteristic equation is

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0.$$

5) Find the characteristic polynomial of

$$\begin{bmatrix} 6 & -2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

[polynomial don't write = 0.

The characteristic polynomial of

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$$

S_1 = Sum of the diagonal elements

$= 6+3+3 = 12$

S_2 = Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$$

$= 8 + 14 + 14 = 36$

$S_3 = |A|$

$= 6(8) + 2(-6+2) + 2(2-6)$

$= 48 - 8 - 8$

$S_3 = 32$

∴ The characteristic polynomial is

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32$$

↳ defn The determinant $|A-\lambda I|$ when expanded will give a polynomial, which $=0$ is called characteristic polynomial of A

Eigen Value :

The roots of the characteristic equation of the matrix are called Eigen Value or Characteristic Value.