SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
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COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT II
ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

Transformation to principal axes :

Conic section :

(1) Find out what type of conic section the following quadratic form represents :

(i) $Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$

(ii) $Q = 3x_1^2 + 22x_1x_2 + 3x_2^2 = 0$

(iii) $Q = x_1^2 - 12x_1x_2 + x_2^2 = 70$.

Solution :

(i) $Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128 \longrightarrow$ ①

$$A = \begin{bmatrix} 17 & -15 \\ -15 & 17 \end{bmatrix}$$

$c_1 = 17 + 17 = 34$

$c_2 = 289 - 225 = 64$

The characteristic equation is,

$$\lambda^2 - c_1\lambda + c_2 = 0$$

$$\lambda^2 - 34\lambda + 64 = 0$$

$$\lambda = 32, 2$$

To find the eigen vectors :

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 17-\lambda & -15 \\ -15 & 17-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow ②$$

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Case (i) : $\lambda = 32$

② $\Rightarrow$ $\begin{pmatrix} 17-32 & -15 \\ -15 & 17-32 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -15 & -15 \\ -15 & -15 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-15 x_1 - 15 x_2 = 0$

$-15 x_1 = 15 x_2$

$-x_1 = x_2$

$\dfrac{x_1}{1} = \dfrac{x_2}{-1}$

$\therefore X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Case (ii) : $\lambda = 2$

② $\Rightarrow$ $\begin{pmatrix} 17-2 & -15 \\ -15 & 17-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 15 & -15 \\ -15 & 15 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$15 x_1 - 15 x_2 = 0$

$15 x_1 = 15 x_2$

$x_1 = x_2$

$\dfrac{x_1}{1} = \dfrac{x_2}{1}$

$X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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The modal matrix,

$$M = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

The normalized matrix,

$$N = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$N^T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$N^T A N = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 17 & -15 \\ -15 & 17 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 32 & 0 \\ 0 & 2 \end{pmatrix} = D$$

$$\therefore N^T A N = D$$

Now $Y^T D Y = \begin{cases} y_1 \\ y_2 \end{cases}$

$$= (y_1 \quad y_2) \begin{pmatrix} 32 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= 32 y_1^2 + 2 y_2^2$$

Given : $Q = 128 \Rightarrow 32 y_1^2 + 2 y_2^2 = 128$

$$\frac{32 y_1^2}{128} + \frac{2 y_2^2}{128} = 1$$

$$\frac{y_1^2}{4} + \frac{y_2^2}{64} = 1$$

$$\Rightarrow \frac{y_1^2}{2^2} + \frac{y_2^2}{8^2} = 1 \quad \text{which represents the ellipse.}$$

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(ii) $Q = 14y_1^2 - 8y_2^2 = 0$ which represent Pair of straight lines

(iii) $Q = 7y_1^2 - 5y_2^2 = 70$ which is a hyperbola.