SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics

Miller indices, Interplanar Distance

MILLER INDICES: The crystal lattice may be regarded as made up of an infinite set of parallel equidistant planes passing through the lattice points which are known as lattice planes. Miller indices are defined as the reciprocals of the intercepts made by the plane on the three crystallographic axes. In simple terms, the planes passing through lattice points are called '*lattice planes*'.

- For a given lattice, the lattice planes can be chosen in a different number of ways.
- The orientation of planes or faces in a crystal can be described in terms of their intercepts on the three axes. Miller introduced a system to designate a plane in a crystal.
- He introduced a set of three numbers to specify a plane in a crystal.
- This set of three numbers is known as '*Miller Indices*' of the concerned plane.
- Miller indices are set of three possible integers represented as (h, k, l) which is used to designate plane in the crystal, is the reciprocal of the intercept made by the planes on the crystallographic axes.
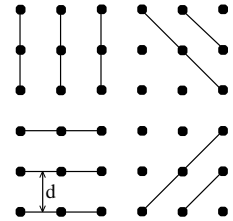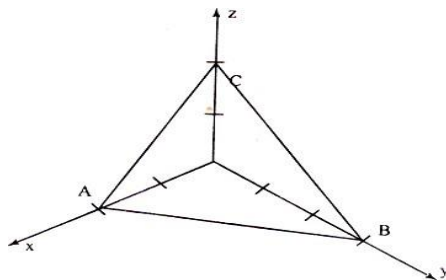

Procedure for finding Miller Indices

Step 1: Determine the intercepts of the plane along the axes X,Y and Z in terms of the lattice constants a,b and c.

Step 2: Determine the reciprocals of these numbers.

Step 3: Find the least common denominator (*lcd*) and multiply each by this *lcd*.

Step 4: The result is written in parenthesis. This is called the `Miller Indices' of the plane in the form (h k l). This is called the `Miller Indices' of the plane in the form (h k l).

Plane ABC has intercepts of 2 units along X-axis, 3 units along Y-axis and 2 units along Z-axis.

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics

DETERMINATION OF 'MILLER INDICES'

Step 1: The intercepts are 2, 3 and 2 on the three axes.

Step 2: The reciprocals are 1/2, 1/3 and 1/2.

Step 3: The least common denominator is '6'. Multiplying each reciprocal by

lcd, we get 3, 2 and 3.

Step 4: Hence Miller indices for the plane ABC is (3 2 3)

IMPORTANT FEATURES OF MILLER INDICES

For the cubic crystal especially, the important features of Miller indices are,

+ A plane which is parallel to any one of the co-ordinate axes has an intercept of infinity (∞).
Therefore the Miller index for that axis is zero; i.e. for an intercept at infinity, the
corresponding index is zero.

EXAMPLE (1 0 0) plane (Plane parallel to Y and Z axes)
+
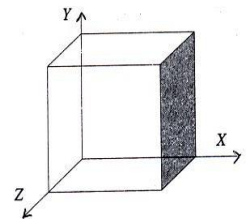
+ In the above plane, the intercept along X axis is 1 unit.
+ The plane is parallel to Y and Z axes. So, the intercepts along Y and Z axes are '∞'.
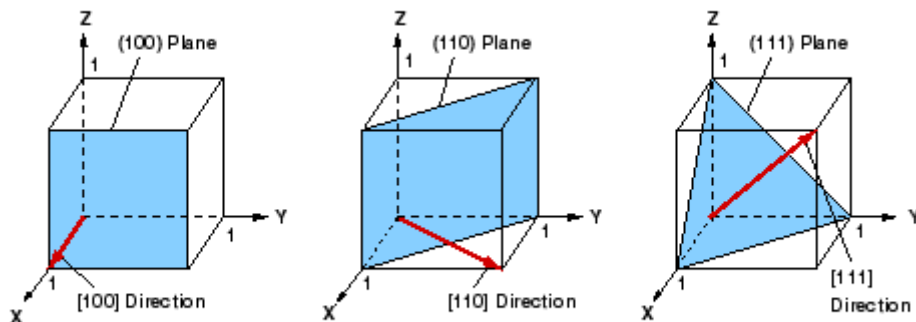+ Now the intercepts are 1, ∞ and∞.
+ The reciprocals of the intercepts are = 1/1, 1/∞ and 1/∞.
+ Therefore the Miller indices for the above plane is (1 0 0).

MILLER INDICES OF SOME IMPORTANT PLANES

SEPARATION BETWEEN LATTICE PLANES

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics

- Consider a cubic crystal of side 'a', and a plane ABC.

- This plane belongs to a family of planes whose Miller indices are (h k l) because Miller indices represent a set of planes.

- Let ON =d, be the perpendicular distance of the plane A B C from the origin.

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics
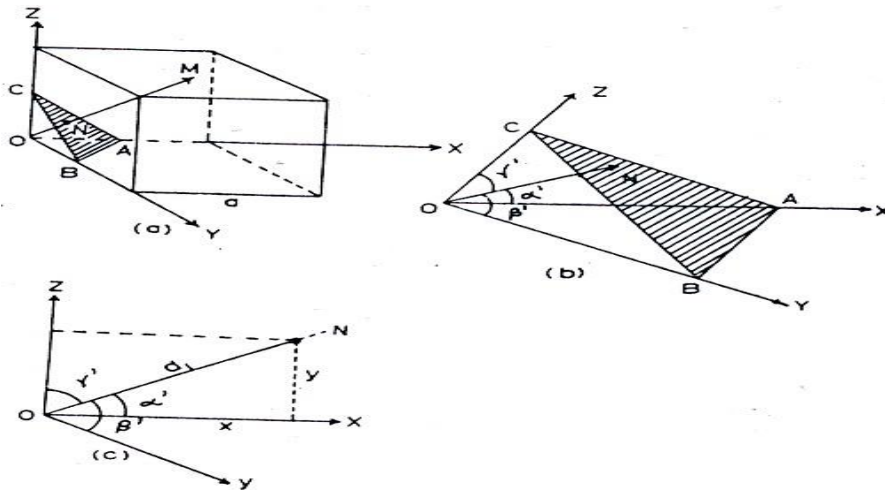
- Let α_1, β_1 and γ_1 (different from the interfacial angles α, β and γ) be the angles between co- ordinate axes X, Y, Z and ON respectively.

- The intercepts of the plane on the three axes are,

$$OA = \frac{a}{h}, \quad OB = \frac{a}{k} \quad \text{and} \quad OC = \frac{a}{l} \longrightarrow (1)$$

From the figure, we have,

$$\cos \alpha^1 = \frac{d_1}{OA}, \cos\beta^1 = \frac{d_1}{OB} \text{ and } \cos\gamma^1 = \frac{d_1}{OC} \longrightarrow$$

(2)

From the property of direction of cosines,

$$\cos^2\alpha^1 + \cos^2\beta^1 + \cos^2\gamma^1 = 1 \longrightarrow (3)$$

Using equation 1 in 2, we get,

$$\cos\alpha^1 = \frac{d_1 h}{a}, \cos\beta^1 = \frac{d_1 k}{a}, \quad \text{and} \quad \cos\gamma^1 = \frac{d_1 l}{a} \longrightarrow (4)$$

Substituting equation (4) in (3), we get,

$$\left(\frac{d_1 h}{a}\right)^2 + \left(\frac{d_1 k}{a}\right)^2 + \left(\frac{d_1 l}{a}\right)^2 = 1 \qquad \frac{d_1^2 h^2}{a} + \frac{d_1^2 k^2}{a} + \frac{d_1^2 l^2}{a} = 1$$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics

$$\frac{\frac{a^2}{d_1}}{a^2}(h^2 + k^2 + l^2) = 1 \qquad \frac{\frac{a^2 d^2}{1}}{a^2}(h^2 + k^2 + l^2) = 1$$

$$d_1^2 = \frac{a^2}{(h^2 + k^2 + l^2)} \qquad d_1 = ON = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \longrightarrow (5)$$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
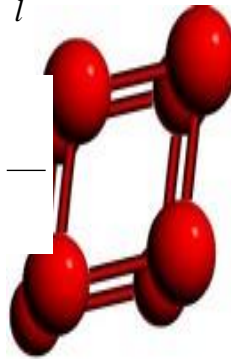Coimbatore – 641 035.

Department of Physics

i.e. the perpendicular distance between the origin and the 1st plane ABC is,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

- Now, let us consider the next parallel plane.

- Let OM=d_2 be the perpendicular distance of this plane from the origin.

- The intercepts of this plane along the three axes are

- $OA' = \frac{2a}{h}, OB' = \frac{2a}{k}, OC' = \frac{2a}{l}$ $\qquad OM = d_2 = \frac{2a}{\sqrt{h^2 + k^2 + l^2}}$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore – 641 035.

Department of Physics

- Therefore, the interplanar spacing between two adjacent parallel planes of Miller indices(h k l) is given by, NM = OM – ON i.e.interplanar spacing

$$d_2 - d_1 = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$