3) What is the no of females in each class after 2, 4, 6 yrs if each class initially consists of 400 females. Let the Leslie matrix be $\begin{bmatrix} 0 & 6 & 0 \\ 0.375 & 0 & 0 \\ 0 & 0.15 & 0 \end{bmatrix}$

Solution:-

Initially, $X_{(0)}^T = [400 \quad 400 \quad 400]$

After 2yrs, $X_{(2)} = L X_{(0)} = \begin{bmatrix} 0 & 6 & 0 \\ 0.375 & 0 & 0 \\ 0 & 0.15 & 0 \end{bmatrix} \begin{bmatrix} 400 \\ 400 \\ 400 \end{bmatrix}$

$= \begin{bmatrix} 2400 \\ 150 \\ 60 \end{bmatrix}$

$\therefore X_{(2)}^T = [2400 \quad 150 \quad 60]$

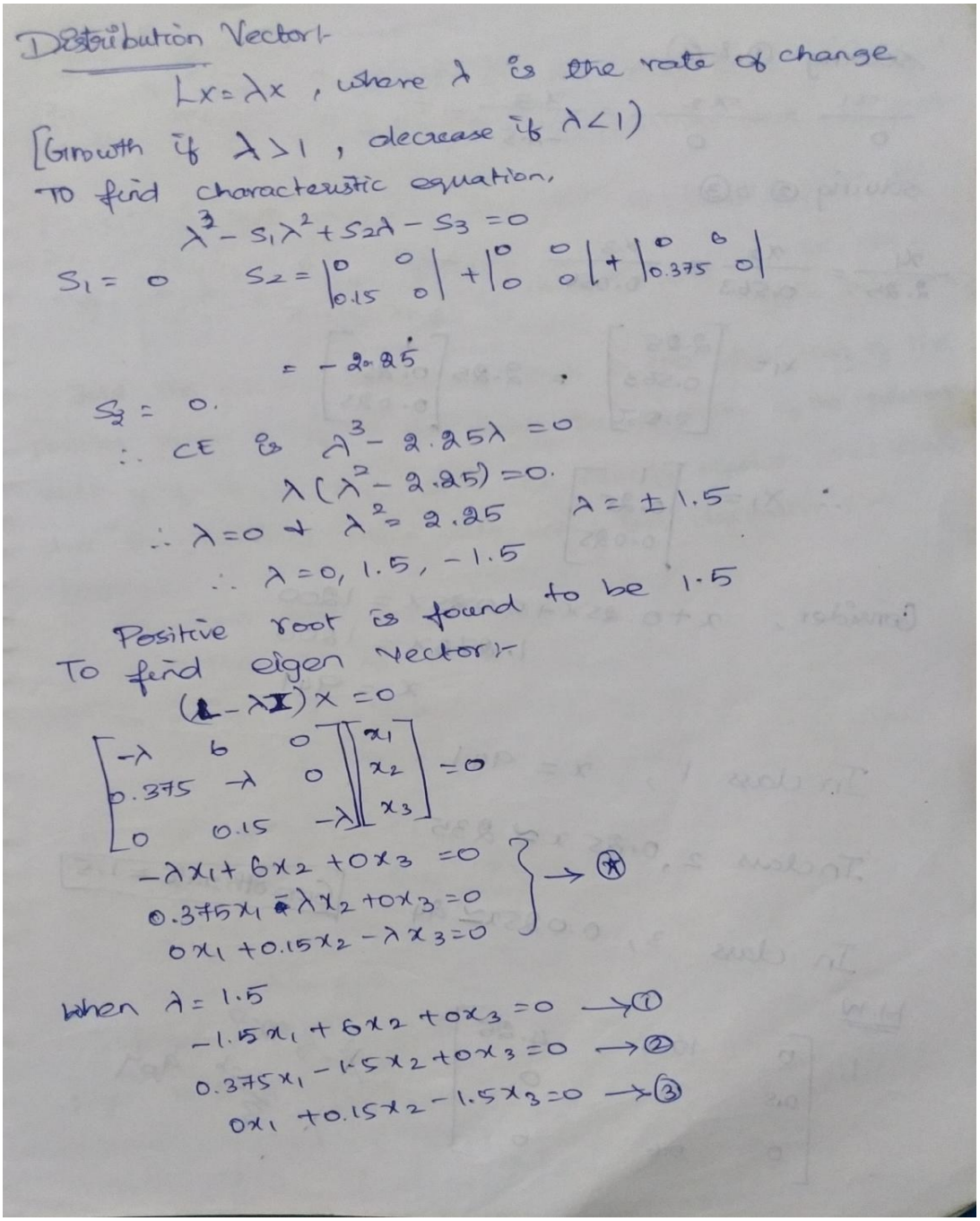
After 4yrs, $X_{(4)} = L X_{(2)} = \begin{bmatrix} 0 & 6 & 0 \\ 0.375 & 0 & 0 \\ 0 & 0.15 & 0 \end{bmatrix} \begin{bmatrix} 2400 \\ 150 \\ 60 \end{bmatrix}$

$= \begin{bmatrix} 900 \\ 900 \\ 22.5 \end{bmatrix}$

$X_4^T = [900 \quad 900 \quad 22.5]$

After 6 yrs, $X_6 = L X_4 = \begin{bmatrix} 0 & 6 & 0 \\ 0.375 & 0 & 0 \\ 0 & 0.15 & 0 \end{bmatrix} \begin{bmatrix} 900 \\ 900 \\ 22.5 \end{bmatrix}$

$= \begin{bmatrix} 5400 \\ 337.5 \\ 135 \end{bmatrix}$



Distribution Vector:-

$Lx = \lambda x$, where λ is the rate of change

[Growth if $\lambda > 1$, decrease if $\lambda < 1$)

To find characteristic equation,

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 0$ $\quad S_2 = \begin{vmatrix} 0 & 0 \\ 0.15 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 6 \\ 0.375 & 0 \end{vmatrix}$

$= -2.25$

$S_2 = 0.$

$\therefore$ CE is $\lambda^3 - 2.25\lambda = 0$

$\lambda(\lambda^2 - 2.25) = 0.$

$\therefore \lambda = 0$ (or) $\lambda^2 = 2.25$ $\quad \lambda = \pm 1.5$

$\therefore \lambda = 0, 1.5, -1.5$

Positive root is found to be 1.5

To find eigen vector:-

$(A - \lambda I)X = 0$

$$\begin{bmatrix} -\lambda & 6 & 0 \\ 0.375 & -\lambda & 0 \\ 0 & 0.15 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-\lambda x_1 + 6x_2 + 0x_3 = 0$

$0.375x_1 - \lambda x_2 + 0x_3 = 0$ $\quad \Big\} \rightarrow (*)$

$0x_1 + 0.15x_2 - \lambda x_3 = 0$

When $\lambda = 1.5$

$-1.5x_1 + 6x_2 + 0x_3 = 0 \rightarrow (1)$

$0.375x_1 - 1.5x_2 + 0x_3 = 0 \rightarrow (2)$

$0x_1 + 0.15x_2 - 1.5x_3 = 0 \rightarrow (3)$

Solving ① & ②

$$\frac{x_1}{0} = \frac{x_2}{0} = \frac{x_3}{-4.5}$$

x_1	x_2	x_3	
6	0	-1.5	6
-1.5	0	0.375	1.5

Solving ② & ③

$$\frac{x_1}{2.25} = \frac{x_2}{0.563} = \frac{x_3}{0.056}$$

x_1	x_2	x_3	
-1.5	0	0.375	-1.5
0.15	-1.5	0	0.15

$$\therefore X_1 = \begin{bmatrix} 2.25 \\ 0.563 \\ 0.056 \end{bmatrix} = 2.25 \begin{bmatrix} 1 \\ 0.250 \\ 0.025 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} 1 \\ 0.25 \\ 0.025 \end{bmatrix}$$

Consider, $x + 0.25x + 0.025x = 1200$

$$1.275x = 1200$$

$$x = 941$$

In class 1, $x = 941$.

In class 2, $0.25x \simeq 235$

In class 3, $0.025x \simeq 24$

Growth rate = 1.5