

UNIT III
DIFFERENTIAL CALCULUS

Problems :

- ① Find the radius of curvature at any point of the Catenary $y = c \cosh(x/c)$.

Soln :

$$y = c \cosh(x/c) \rightarrow \textcircled{1}$$

$$y_1 = c \cdot \sinh(x/c) \cdot \frac{1}{c} = \sinh(x/c)$$

$$y_2 = \cosh(x/c) \cdot \frac{1}{c}$$

$$P = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1 + \sinh^2(x/c))^{3/2}}{\cosh(x/c)} \cdot c$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$= \frac{(\cosh^2(x/c))^{3/2}}{\cosh(x/c)} \cdot c$$

$$P = c \cosh^2(x/c) \rightarrow \textcircled{2}$$

From $\textcircled{1}$, $\cosh(x/c) = y/c$

subs this in $\textcircled{2}$, we get,

$$P = c \cdot \frac{y^2}{c^2}$$

$$\boxed{P = y^2/c}$$

(2) Find ρ for (i) $y = \log \sin x$ (ii) $y = c \log \sec (x/c)$

Soln:

(i) $y = \log \sin x$

$$y_1 = \frac{1}{\sin x} \cos x = \cot x$$

$$y_2 = -\operatorname{cosec}^2 x$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + \cot^2 x)^{3/2}}{-\operatorname{cosec}^2 x}$$

$$= \frac{\operatorname{cosec}^3 x}{-\operatorname{cosec}^2 x} = -\operatorname{cosec} x$$

$$\boxed{|\rho| = \operatorname{cosec} x}$$

(ii) $y = c \log \sec (x/c)$

$$y_1 = c \frac{1}{\sec (x/c)} \sec \left(\frac{x}{c}\right) \tan \left(\frac{x}{c}\right) \cdot \frac{1}{c}$$

$$y_1 = \tan (x/c)$$

$$y_2 = \frac{1}{c} \sec^2 \left(\frac{x}{c}\right)$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + \tan^2 (x/c))^{3/2}}{\frac{1}{c} \sec^2 (x/c)}$$

$$\rho = c \frac{\sec^3 (x/c)}{\sec^2 (x/c)} \Rightarrow \boxed{\rho = c \sec (x/c)}$$



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3 Find the radius of curvature for the curves

(i) $y = e^x$, where it crosses the y-axis

(ii) $y^2 = x^3 + 8$ at the point $(-2, 0)$

(iii) $xy = c^2$ at $x = c$

(iv) $y^2 = \frac{a^3 - x^3}{x}$ at $(a, 0)$

(v) $y = \frac{\log x}{x}$ at $x = 1$

(vi) $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$

Soln:

(i) $y = e^x$

On y-axis $x = 0$

when $x = 0$, $y = e^0 = 1 \therefore y = 1$

\therefore The point is $(0, 1)$

$$y = e^x$$

$$y_1 = e^x ; y_1 \text{ at } (0, 1) = 1$$

$$y_2 = e^x ; y_2 \text{ at } (0, 1) = 1$$

$$\therefore P = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + 1)^{3/2}}{1} = 2^{3/2}$$

$$P = 2\sqrt{2}$$



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$$(i) y^2 = x^3 + 8$$

Diff w.r.t 'x', we get

$$2y y_1 = 3x^2$$

$$y_1 = \frac{3x^2}{2y} \quad ; \quad y_1(-2, 0) = \infty$$

$$\therefore \frac{dx}{dy} = 0 \quad ; \quad \frac{dx}{dy} = \frac{2y}{3x^2}$$

$$\frac{d^2x}{dy^2} = \frac{(3x^2) \cdot 2 - (2y) \cdot 6x \cdot \frac{dx}{dy}}{(9x^4)}$$

$$= \frac{6x^2 - 12xy \left(\frac{dx}{dy}\right)}{9x^4}$$

$$\left(\frac{d^2x}{dy^2}\right)_{(-2,0)} = \frac{6x^2}{9x^4 \cdot \frac{1}{x^2}} = \frac{1}{6}$$

$$P = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}}{d^2x/dy^2} = \frac{(1+0)^{3/2}}{1/6}$$

$$P = 6$$

$$(ii) xy = c^2 \text{ at } x = c$$

If $x = c$, $xy = c^2$ will be



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$$y = c^2/x = c^2/c = c$$

$$y = c$$

∴ The point is (c, c) .

$$xy = c^2$$

$$x y_1 + y = 0$$

$$y_1 = -y/x$$

$$y_1 \text{ at } (c, c) \text{ is } = -c/c = -1$$

$$y_2 = \frac{x(-dy/dx) - (-y) \cdot 1}{x^2}$$

$$= \frac{-x \times (-y/x) + y}{x^2} = \frac{2y}{x^2}$$

$$y_2 \text{ at } (c, c) = \frac{2c}{c^2} = \frac{2}{c}$$

$$\therefore \rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1 + (-1)^2)^{3/2}}{2/c} = \frac{2^{3/2} c}{2}$$

$$= \frac{2\sqrt{2} c}{2}$$

$$\boxed{\rho = \sqrt{2} \cdot c}$$