(3) $\frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2y(t) = x(t)$

$s^2 Y(s) - s Y(s) - 2 Y(s) = X(s)$

$(s^2 - s - 2) Y(s) = X(s)$

$\therefore \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$

$\frac{A}{s-2} + \frac{B}{s+1} = \frac{1}{(s-2)(s+1)}$

$1 = A(s+1) + B(s-2)$

$s = -1$

$B(-3) = 1$

$\boxed{B = -1/3}$

put $s = 2$

$3A = 1$

$\boxed{A = 1/3}$

$H(s) = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}$

$= \frac{1}{3} L^{-1}\left(\frac{1}{s-2}\right) - \frac{1}{3} L^{-1}\left(\frac{1}{s+1}\right)$

$h(t) = \frac{1}{3} e^{2t} u(t) - \frac{1}{3} e^{-t} u(t)$

(4) $H(s) = \frac{s}{s^2+5s+6}$ and $x(t) = e^{-t} u(t)$. Determine the o/p assuming zero Initial conditions.

$Y(s) = \frac{s}{s^2+5s+6} \qquad X(s) = \frac{1}{s+1}$

$Y(s) = \frac{s}{(s+3)(s+2)(s+1)}$

$$S = A(S+2)(S+1) + B(S+3)(S+1) + C(S+3)(S+2)$$

Put $S=-2$: $-2 = B(1)(-1)$, $B=2$

Put $S=-3$: $-3 = 2A$, $A = -3/2$

Put $S=-1$: $-1 = C(2)(1)$, $C = -1/2$

$$Y(S) = \frac{-3}{2(S+3)} + \frac{2}{S+2} - \frac{1}{2(S+1)}$$

$$= -3/2\, L^{-1}\left(\frac{1}{S+3}\right) + 2\, L^{-1}\left(\frac{1}{S+2}\right) - \frac{1}{2}\, L^{-1}\left(\frac{1}{S+1}\right)$$

$$y(t) = -3/2\, e^{-3t}\, u(t) + 2\, e^{-2t}\, u(t) - 1/2\, e^{-t}\, u(t)$$

(5) Find the Impulse response of the system :-

$$RC\, d/dt\, y(t) + y(t) = x(t)$$

$$RC\, sY(S) + Y(S) = X(S)$$

$$Y(S)(RCS+1) = X(S)$$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{1}{(RCS+1)}$$

$$y(t) = L^{-1}\left[\frac{1}{RCS+1}\right]$$

$$= 1/RC\left[L^{-1}\left(\frac{1}{S+1/RC}\right)\right]$$

$$\therefore y(t) = \frac{1}{RC}\, e^{-t/RC}\, u(t)$$

(6) Find the o/p of the system :- $h(t) = u(t)$, $x(t) = e^{-2t} \cdot u(t)$

$$H(S) = 1/S \qquad X(S) = \frac{1}{S+2}$$

$$H(S) = \frac{Y(S)}{X(S)}$$

$$Y(S) = H(S) \cdot X(S)$$

$$= 1/S \cdot \frac{1}{S+2}$$

$$= \frac{A}{S} + \frac{B}{S+2} \Rightarrow A(S+2) + B(S)$$

$S = 0$ $\qquad$ $S = -2$

$1 = A(2)$ $\qquad$ $1 = A(0) + B(-2)$

$\boxed{A = 1/2}$ $\qquad$ $\boxed{B = -1/2}$

$$Y(S) = \frac{1}{2(S)} - \frac{1}{2(S+2)}$$

$$= 1/2\, L^{-1}\left(1/S\right) - 1/2\, L^{-1}\left(\frac{1}{S+2}\right)$$

$$y(t) = 1/2\, u(t) - 1/2\, e^{-2t}\, u(t)$$