① Find the Inverse Laplace Transform of $x(S) = \frac{S^2+2S-2}{S(S+2)(S-3)}$ using partial fraction Method.

$$\frac{S^2+2S-2}{S(S+2)(S-3)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S-3}$$

$$S^2+2S-2 = A(S+2)(S-3) + B(S)(S-3) + C(S)(S+2)$$

when $S=0$

$$-2 = -6A$$

$$\boxed{A = 1/3}$$

when $S=-2$

$$(-2)^2 + 2(-2) - 2 = B(-5)(-2)$$

$$4-4-2 = 10B$$

$$\boxed{B = -1/5}$$

when $S=3$

$$9+6-2 = 15C$$

$$13 = 15C$$

$$\boxed{C = 13/15}$$

$$\frac{S^2+2S-2}{S(S+2)(S-3)} = \frac{1/3}{S} - \frac{1/5}{S+2} + \frac{13/15}{S-3}$$

$$= \frac{1}{3} L^{-1}\left(\frac{1}{S}\right) - \frac{1}{5} L^{-1}\left(\frac{1}{S+2}\right) + \frac{13}{15} L^{-1}\left(\frac{1}{S-3}\right)$$

$$x(t) = \frac{1}{3} u(t) - \frac{1}{5} e^{-2t} u(t) + \frac{13}{15} e^{3t} u(t)$$

② Find the ILT of $x(S) = \frac{1}{S^2+3S+2}$ using Partial fraction Method :- ROC : $-2 < Re(S) < -1$

$$\frac{1}{S^2+3S+2} = \frac{A}{S+2} + \frac{B}{S+1}$$

$$1 = A(S+1) + B(S+2)$$

when $S=-2$ $\boxed{A=-1}$

when $S=-1$ $\boxed{B=1}$

$$\frac{1}{(S+2)(S+1)} = \frac{-1}{S+2} + \frac{1}{S+1}$$

$$= (-1)\, L^{-1}\left(\frac{1}{S+2}\right) + (1)\, L^{-1}\left(\frac{1}{S+1}\right)$$

$$x(t) = -e^{-2t}\, u(t) + e^{-t}\, u(t)$$

ROC : $-2 < Re(S) < -1$

$Re(S) > -2$, $Re(S) < -1$

$$x(t) = \left[e^{-2t}\, u(t)\right] + \left[-e^{-t}\, u(-t)\right]$$

(3) $X(S) = \dfrac{4}{(S+2)(S+4)}$ ROC : (i) $-2 > Re(S) > -4$

(ii) $Re(S) < -4$

$$\frac{4}{(S+2)(S+4)} = \frac{A}{S+2} + \frac{B}{S+4}$$

$$4 = A(S+4) + B(S+2)$$

when $S = -4$

$4 = -2B$

$\boxed{B = -2}$

when $S = -2$

$4 = 2A$

$\boxed{A = 2}$

$$X(S) = \frac{2}{S+2} - \frac{2}{S+4}$$

$$= 2\, L^{-1}\left(\frac{1}{S+2}\right) - 2\, L^{-1}\left(\frac{1}{S+4}\right)$$

$$x(t) = 2e^{-2t}\, u(t) - 2e^{-4t}\, u(t)$$

RoC: $-2 > Re(S) > -4$

$Re(S) > -4$, $Re(S) < -2$

$$x(t) = \left[2e^{-4t}\, u(t)\right] + \left[2e^{-2t}\, u(-t)\right]$$

4 HW $X(S) = \frac{3S+7}{S^2-2S-3}$ (i) $Re(S) > 3$ (ii) $Re(S) < -1$ (iii) $-1 < Re(S) < 3$

5 $X(S) = \frac{-3}{(S+2)(S-1)}$

$$\frac{-3}{(S+2)(S-1)} = \frac{A}{S+2} + \frac{B}{S-1}$$

$$-3 = A(S-1) + B(S+2)$$

when $S=1$

$-3 = 3B$

$\boxed{B=1}$

when $S=-2$

$-3 = -3A$

$\boxed{A=1}$

$$X(S) = \frac{1}{S+2} + \frac{1}{S+1}$$

$$= L^{-1}\left(\frac{1}{S+2}\right) + L^{-1}\left(\frac{1}{S+1}\right)$$

$$x(t) = e^{-2t}\, u(t) + e^{-t}\, u(t)$$

(i) $-2 < Re(S) < 1$

$Re(S) > -2$ $Re(S) < 1$

$$x(t) = e^{-2t}\, u(t) - e^{-t}\, u(-t)$$

(ii) $Re(S) > 1$

$$x(t) = e^{-t}\, u(t)$$

(iii) $Re(S) < -2$

$$x(t) = -1\, e^{-2t}\, u(-t)$$