SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT202 – SIGNALS AND SYSTEMS

II YEAR/ III SEMESTER

UNIT 3 – LTI CONTINUOUS TIME SYSTEMS

TOPIC – **LTI SYSTEMS USING FOURIER TRANSFORM**

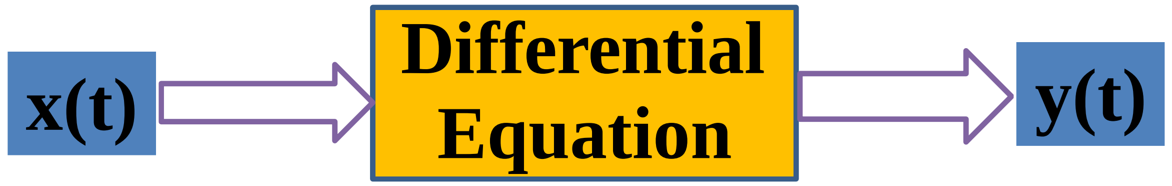
LTI SYSTEM

- Linear Time Invariant Systems (LTI) are characterized with the help of

1. Differential Equation
2. Impulse Response
3. Block Diagrams
4. State Variable description
5. Transfer Functions

Differential Equation :

- It is used to represent continuous time linear time invariant system
- It relates the input and output of the system



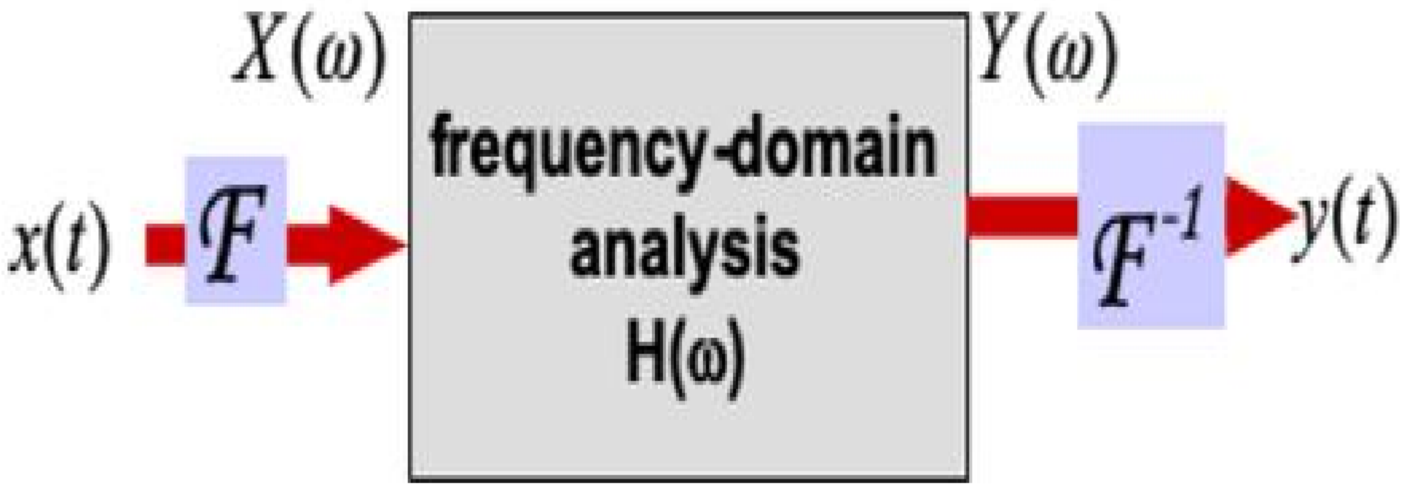
LTI SYSTEM

- **System Transfer Function:** Ratio of the output to the input.

$$H(s) = \frac{Y(s)}{X(s)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



LTI SYSTEM

- Condition for an Linear Time Invariant (LTI) system to be causal:

$$h(t) = 0, \; t<0$$

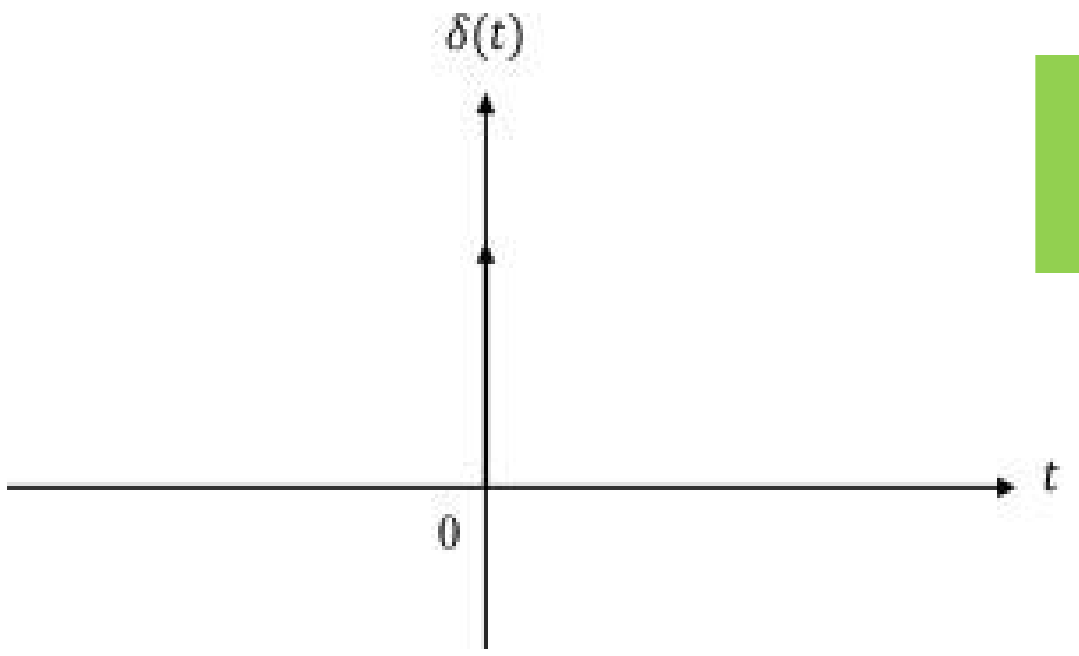
- Condition for an Linear Time Invariant (LTI) system to be stable:

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

IMPULSE RESPONSE

- Impulse response is the output generated by the system, when an unit impulse is applied at the input.

$x(t) = \delta(t)$ → **LTI System** → $y(t) = h(t)$



$$\delta(t) = 1 \text{ for } t = 0$$
$$= 0 \text{ for } t \neq 0$$

- $H(s) = \dfrac{Y(s)}{X(s)}$
- $h(t) = L^{-1}\{H(s)\}$

FOURIER TRANSFORM

- Fourier transform can be applied for both periodic and non periodic signals
- It can be represented in frequency domain
- It provides effective reversible transformation link between frequency domain and time domain representation of the signal
- The spacing between spectral components becomes infinitesimal and hence the frequency spectrum appears to be continuous
- Periodic signals has fixed period To

FOURIER TRANSFORM

General Transform as problem-solving tool

Analysis

Time, t → **Frequency, f**

F

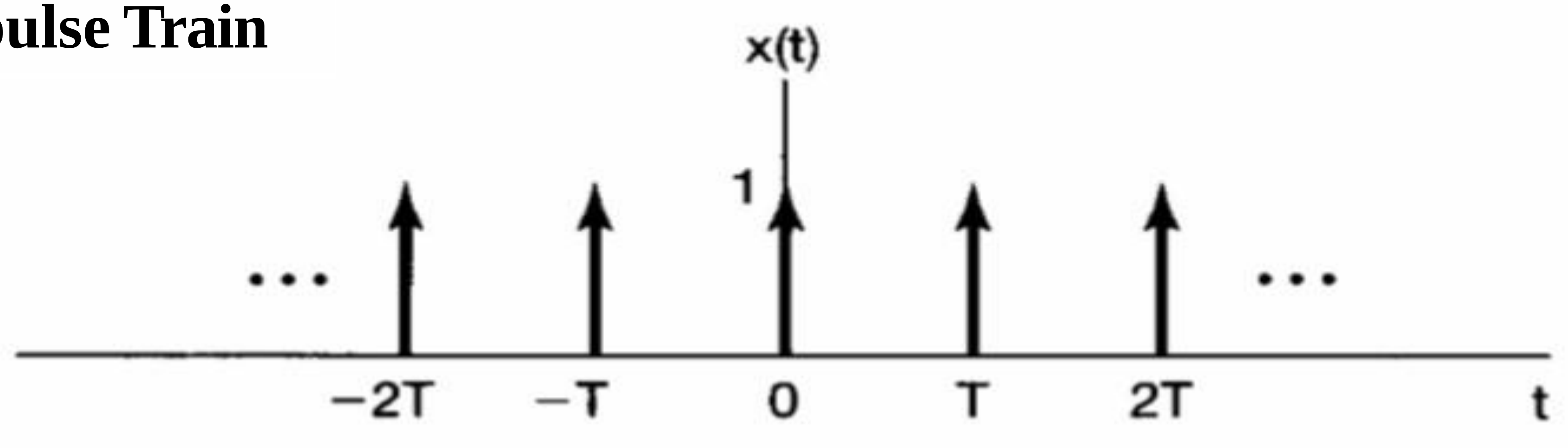
s(t) → **S(f) = F[s(t)]**

Synthesis

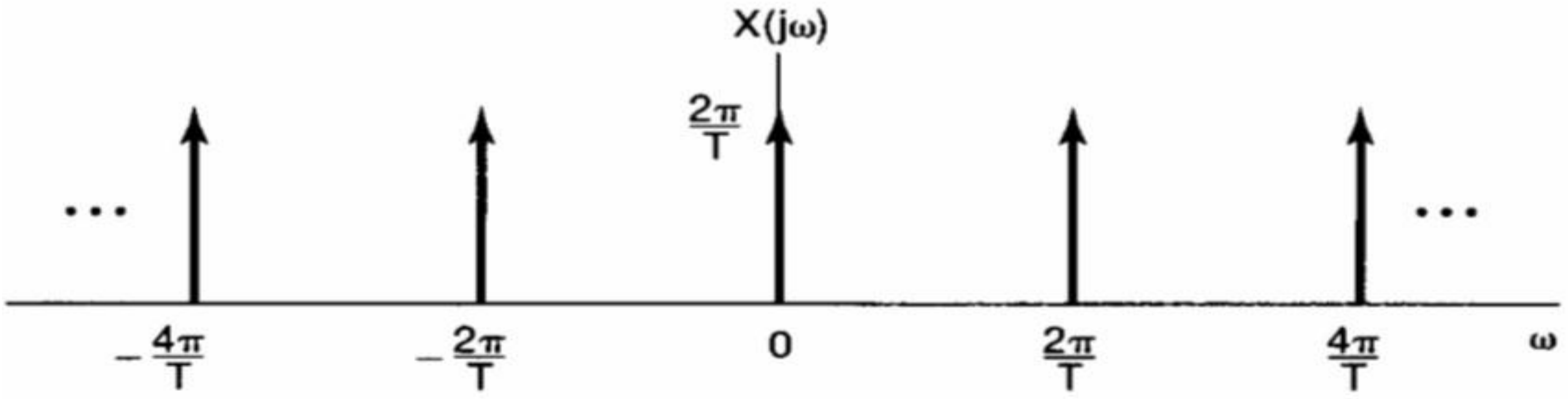
s(t), S(f) : Transform Pair

REPRESENTATION OF FT

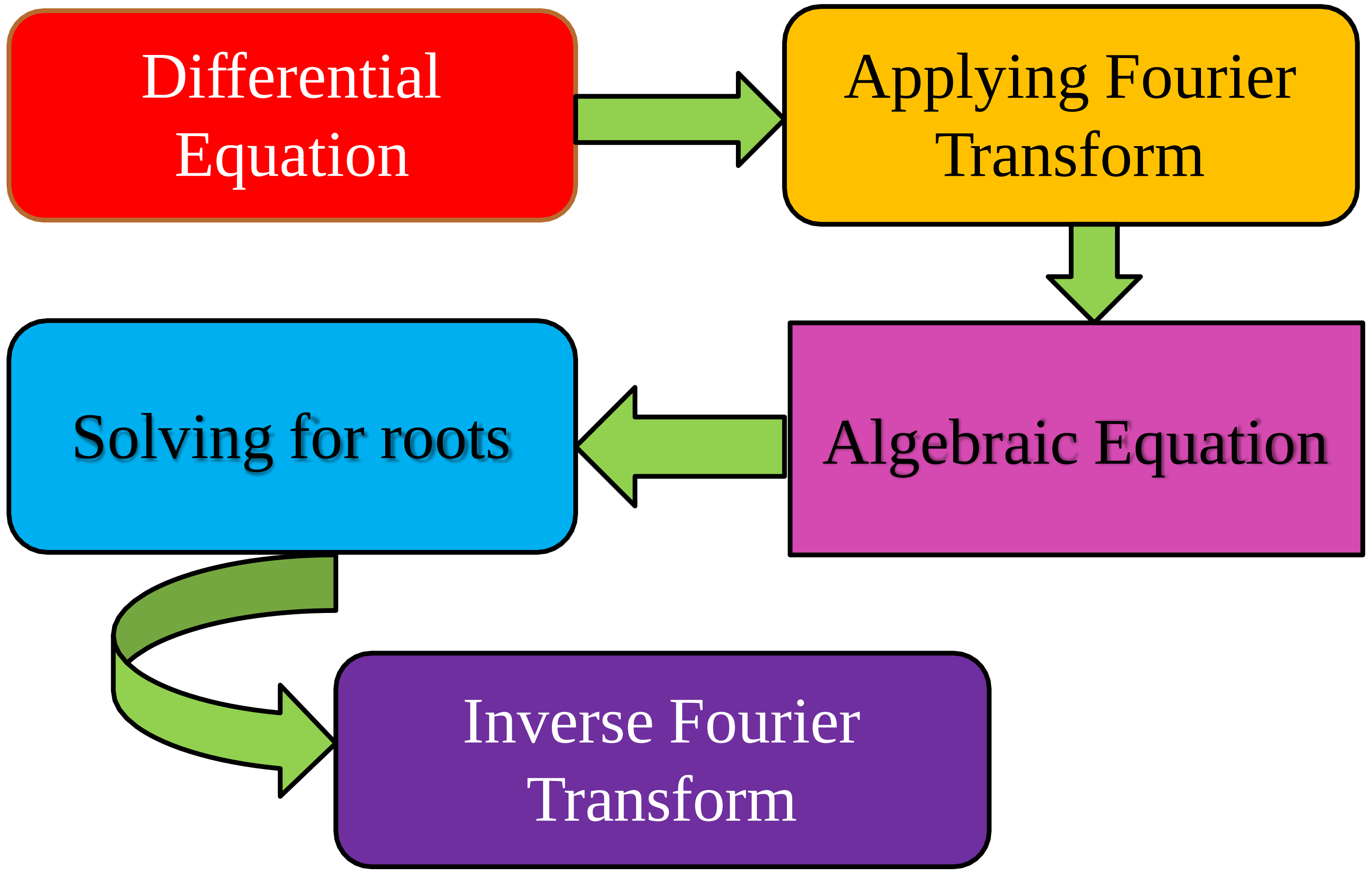
Periodic Impulse Train



Its Fourier Transform

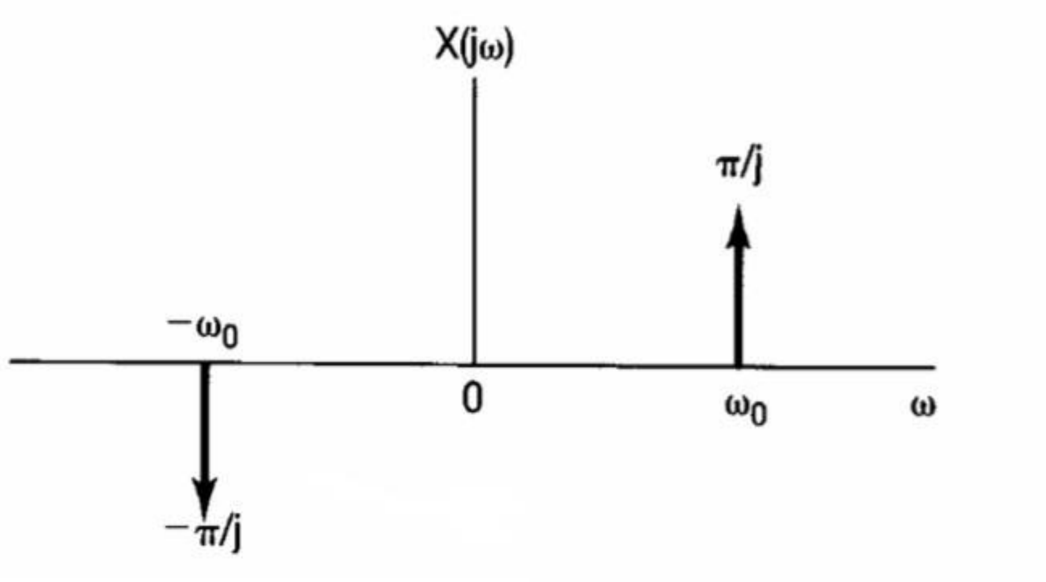


TO FIND IMPULSE RESPONSE

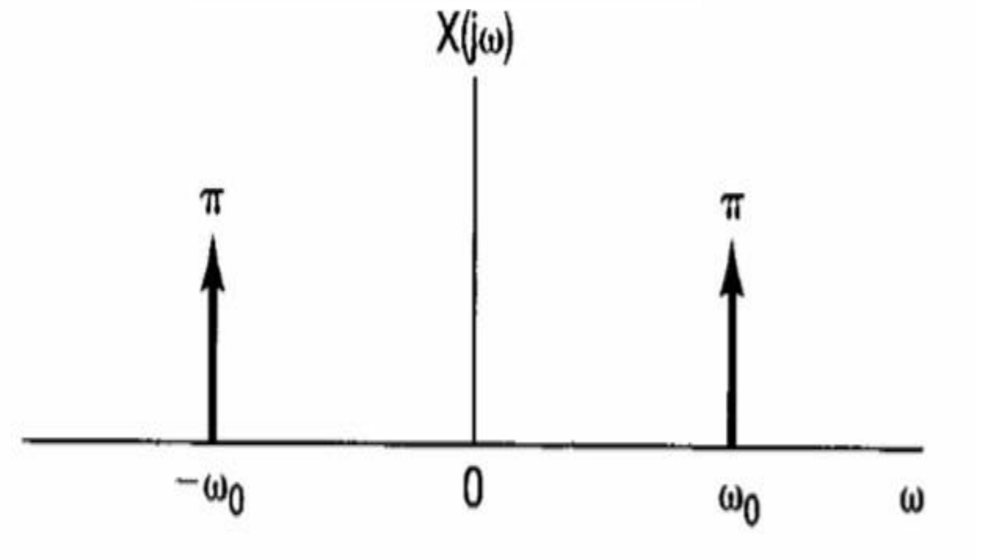


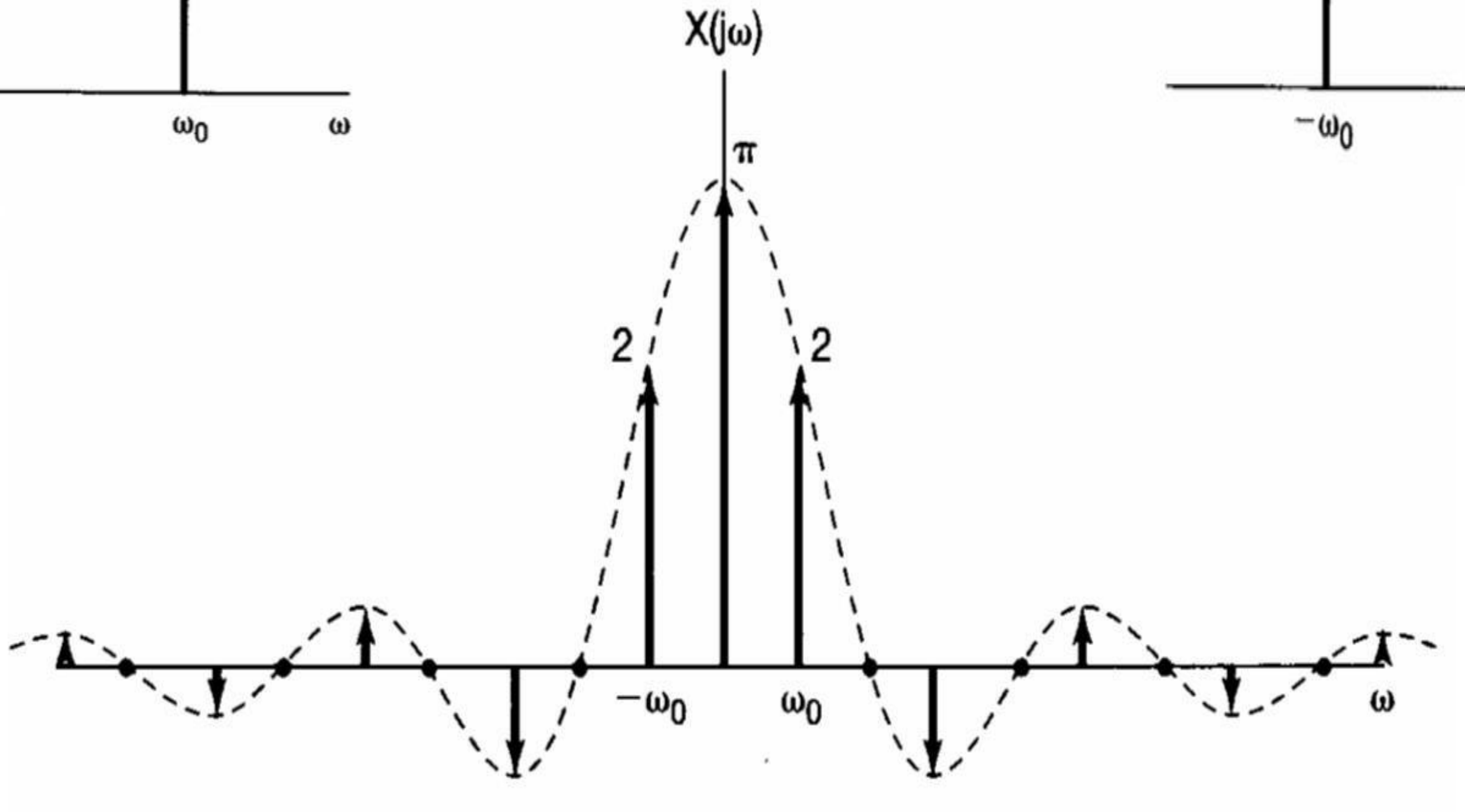
PERIODIC SYMMETRIC SQUARE WAVE

$X(t) = \sin \omega_o t$



$X(t) = \cos \omega_o t$





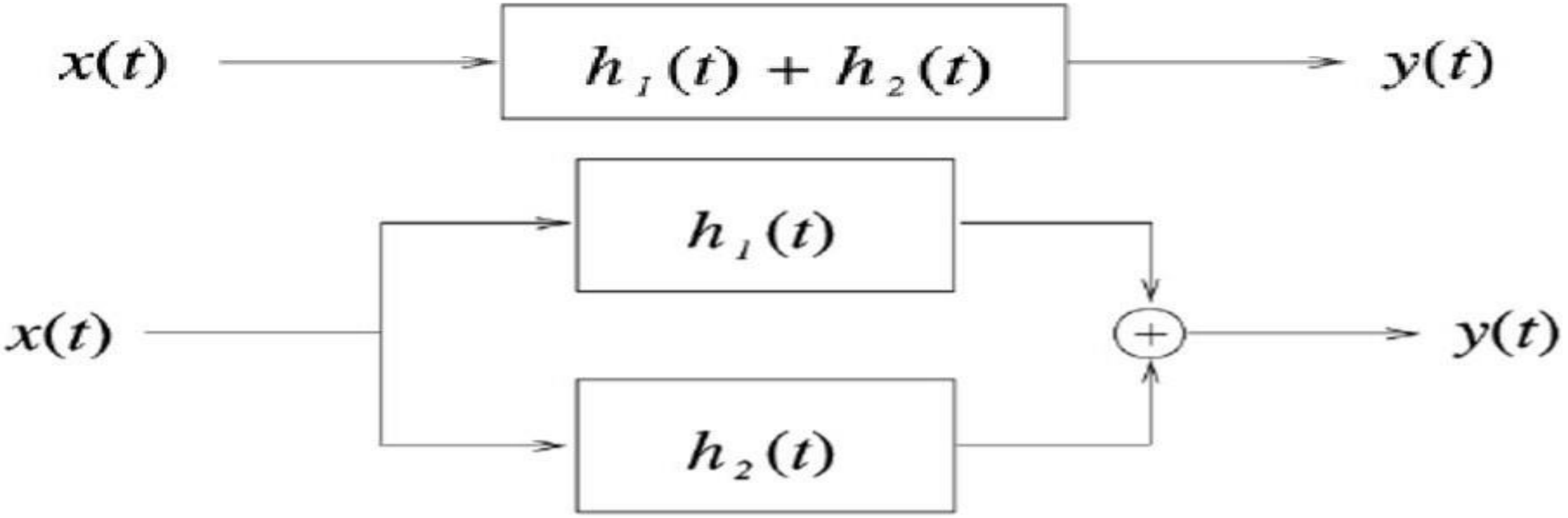
PROPERTIES OF CONVOLUTION INTEGRAL

COMMUTATIVE

$$x(t) * h(t) = h(t) * x(t)$$

DISTRIBUTIVE

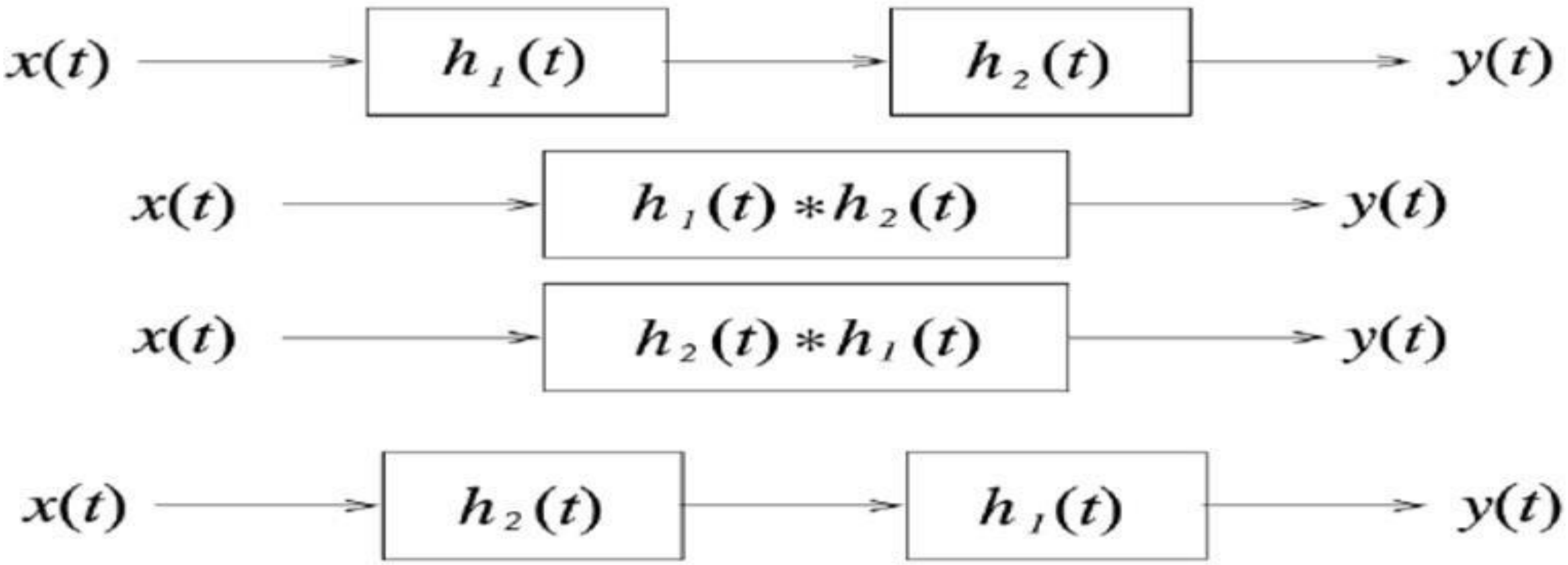
$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$



ASSOCIATIVE PROPERTY

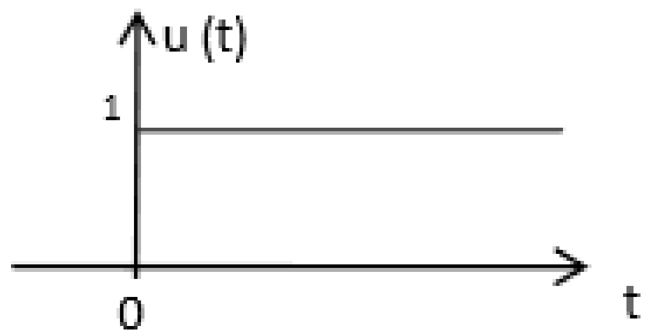
$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

$$[x(t) * h_2(t)] * h_1(t) = x(t) * [h_2(t) * h_1(t)]$$

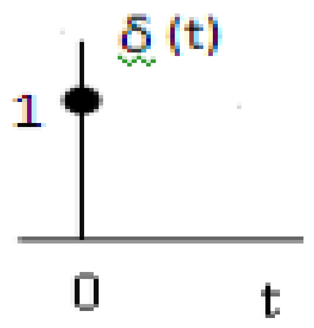


BASIC SIGNALS

$$u(t) = 1 \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$



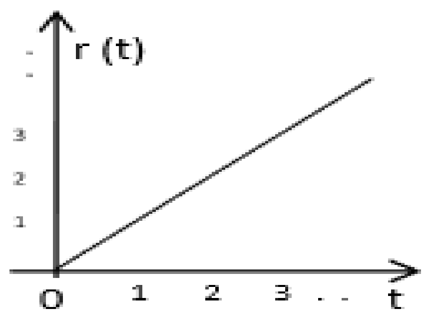
Unit step signal



Unit Impulse signal

$$\delta(t) = 1 \text{ for } t = 0$$
$$= 0 \text{ for } t \neq 0$$

$$r(t) = t \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$

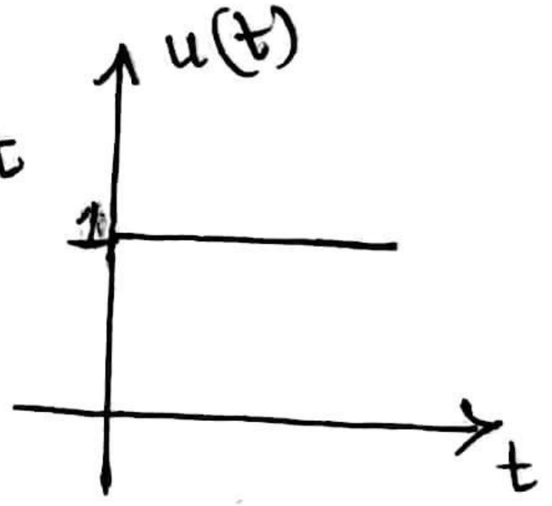


Unit ramp signal

FOURIER TRANSFORM RESULTS

Fourier transform of $x(t) = u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$



$$= \int_{0}^{\infty} (1)\, e^{-j2\pi f t}\, dt$$

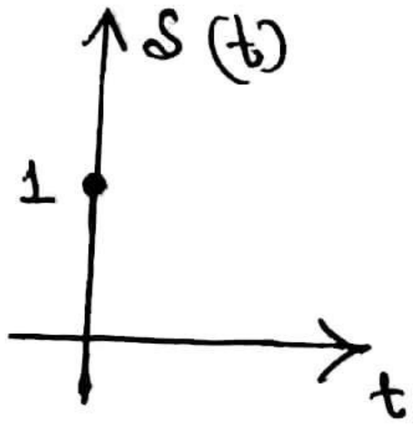
$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_0^{\infty}$$

$$\boxed{X(F) = \frac{1}{j2\pi f}}$$

Fourier transform of Unit Impulse function

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\boxed{X(F) = 1}$$



FOURIER TRANSFORM RESULTS

The System produces the o/p - $y(t) = e^{-t} u(t)$ for an i/p $x(t) = e^{-2t} u(t)$

Determine the Impulse response & freq response of the s/m.

$y(t) = e^{-t} u(t)$

$$Y(F) = \frac{1}{1+j2\pi F}$$

$x(t) = e^{-2t} u(t)$

$$X(F) = \frac{1}{2+j2\pi f}$$

Freq Response :-

$$H(F) = \frac{Y(F)}{X(F)} = \frac{2+j2\pi F}{1+j2\pi F}$$

Impulse Response :-

$$h(t) = F^{-1}[H(F)]$$

$$= F^{-1}(1) + F^{-1}\left(\frac{1}{1+j2\pi f}\right)$$

$$\therefore h(t) = \delta(t) + e^{-t} u(t)$$

FOURIER TRANSFORM RESULTS

The Diff equation of the system is given as

$$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 6 y(t) = \frac{-d}{dt} x(t)$$

Determine freq response

By taking Fourier transform

$$(j\omega)^2 Y(\omega) + 5 j\omega Y(\omega) + 6 Y(\omega) = (-j\omega) X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 5j\omega + 6\right] = -j\omega X(\omega)$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$H(\omega) = \frac{-j\omega}{(j\omega+2)(j\omega+3)}$$

ASSESSMENT

1. The system transfer function is given by -------------
2. ------------- relates the input and output of the system.
3. What is meant by impulse response?
4. Define Unit step and Unit Impulse Signal.
5. The condition of an LTI system to be causal is given by -------------
6. Fourier transform of Unit step function is given by ----------------
7. The condition of stability of an LTI system is --------------
8. LTI Systems are characterized with the help of -------------------

THANK YOU