



DEPARTMENT OF MATHEMATICS

23MAT101 - MATRICES AND CALCULUS

UNIT-I MATRIX EIGENVALUE PROBLEM

EIGEN VALUE PROBLEMS

Characteristic Equations

If A is a square matrix of order n , we can form the matrix $A-dI$, where d is a scalar and I is the unit matrix of order n , then $|A-dI| = 0$ is called the characteristic equation.

The determinant $|A-dI|$ when expanded will give a polynomial, which is called as a characteristic polynomial of matrix A .

Note:

1) For any square matrix A , the sum of the eigen values of a matrix is equal to trace of the matrix.

2) For a 2×2 matrix, the characteristic eqn

$$\text{is, } d^2 - c_1 d + c_2 = 0 \quad (d-d_1)(d-d_2)$$

where $c_1 = \text{sum of the main diagonal elements}$

$$c_2 = |A|$$

3) For a 3×3 matrix, the characteristic eqn is,

$$d^3 - c_1 d^2 + c_2 d - c_3 = 0$$

where $c_1 = \text{sum of the main diagonal elements}$

$c_2 = \text{sum of the minors of main diagonal elements.}$

$$c_3 = |A|$$



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Problems:

1) Find the characteristic equation of $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

Sol:

Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

The char eqn is, $d^2 - C_1d + C_2 = 0$

$C_1 =$ sum of the diagonal elements

$$C_1 = 1 + 2 = 3$$

$$C_2 = |A| = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$\therefore d^2 - 3d + 2 = 0$ is the char eqn.

2) Find the char eqn of $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

Sol:

Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

The char eqn is, $d^3 - C_1d^2 + C_2d - C_3 = 0$

$$C_1 = 2 + 1 - 4 = -1$$

$$C_2 = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= (-4 - 6) + (-8 + 5) + (2 + 9) = -10 - 3 + 11 = -2$$

$$C_3 = \begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{vmatrix} = 2(-4 - 6) - (-3)(-12 + 15) + 1(6 + 5)$$
$$= -20 + 9 + 11 = 0$$

\therefore the char eqn is, $d^3 + d^2 - 2d = 0$

3) If the char eqn of $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 2 & 1 \\ -7 & 2 & 3 \end{pmatrix}$ is

$d^3 + ad^2 + bd + 18 = 0$, find the values of a & b .

Sol:

$a =$ sum of the diagonal elements

$$= 2 + 2 + 3$$

$$\boxed{a = 7}$$



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$b =$ Sum of the minors of the diagonal elements

$$= \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ -7 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix}$$

$$= 6 - 2 + 6 - 0 + 4 - 4$$

$$\boxed{b = 10}$$

4) Write the two matrices with $d^2 - 7d + 6 = 0$ as the char equ.

Soln Since $d^2 - 7d + 6 = 0$ is the char equ.

$$\therefore c_1 = 7, c_2 = 6$$

The two matrices are

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix} \quad \& \quad B = \begin{pmatrix} 6 & 0 \\ 4 & 1 \end{pmatrix}$$

5) Find the char polynomial of

(i) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, Soln: $d^2 - 4d - 5$

(ii) $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, Soln: $d^2 - 5d + 7$

6) Find the char equ of

(i) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, Soln: $d^3 - 18d^2 + 45d = 0$

(ii) $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$, Soln: $d^3 - 3d^2 + 2d = 0$

(iii) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, Soln: $d^3 - 7d^2 + 36 = 0$