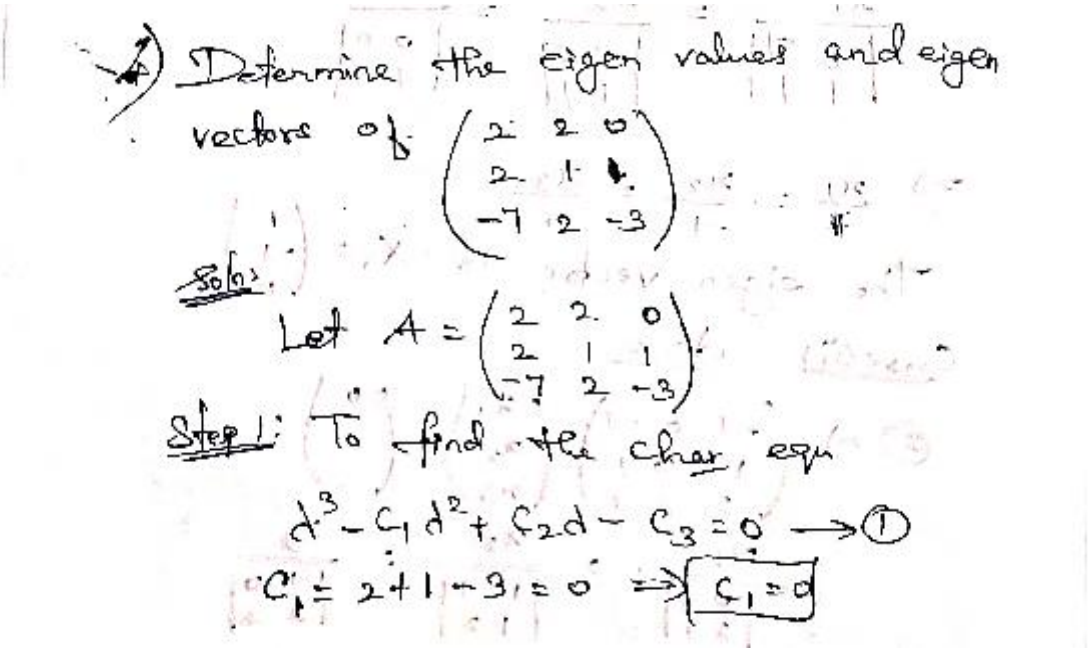

4) Determine the eigen values and eigen vectors of $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

Soln:

Let $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

Step 1: To find the char. eqn

$$d^3 - c_1 d^2 + c_2 d - c_3 = 0 \longrightarrow (1)$$

$$c_1 = 2 + 1 - 3 = 0 \Rightarrow \boxed{c_1 = 0}$$

$$C_2 = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ -7 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (-3-2) + (-6) + (2-4)$$

$$= -5-6-2$$

$$\boxed{C_2 = -13}$$

$$C_3 = \begin{vmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{vmatrix} = 2(-3-2) - 2(-6+7)$$

$$= -10-2$$

$$\boxed{C_3 = -12}$$

$$\lambda^3 - 13\lambda + 12 = 0$$

Step 2: To find the eigen values

$$\lambda^3 - 13\lambda + 12 = 0$$

$$\lambda = 1, 3, -4$$

Step 3: To find the eigen vectors

$$(A - \lambda I)X = 0.$$

$$\begin{pmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow (2)$$

Case (i): $\lambda = 1$

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-4}$$

∴ The eigen vector is $X_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

Case(ii) : $\lambda = -4$

$$\begin{pmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{x_2}{-6} = \frac{x_3}{26}$$

$$\frac{x_1}{1} = \frac{x_2}{-3} = \frac{x_3}{13}$$

$\therefore$ The eigen vector is $X_2 = \begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$

Case(iii) $\lambda = 3$.

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$\therefore$ The eigen vector is $X_3 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

Hence the eigen values & eigen vectors are

λ	1	3	-4
X	$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$

5) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}.$$

Sol:

$c_1 = 7,\ c_2 = 0,\ c_3 = -36$

$\lambda^3 - 7\lambda^2 + 36 = 0$

$\lambda = -2, 3, 6$

$X = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$

6) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}.$$

Sol:

$c_1 = 6,\ c_2 = 11,\ c_3 = 6$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1, 2, 3.$

$X = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$

7) Find the eigen values & eigen vectors of $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

Sol:

$\lambda = 2, 3, 5$

$X = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

8) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$$

Sol:

$\lambda = 0, 1, 2$

$X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}.$

Home work

(1) Find the eigen values and eigen vectors of

(i) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(2) Find the eigen values and eigen vectors of

$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.