



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

23MAT101 - MATRICES AND CALCULUS

UNIT-I MATRIX EIGENVALUE PROBLEM

Q) Determine the eigen values and eigen vectors of

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

Sol:

Let $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

Step 1: To find the char. eqn

$$\lambda^3 - C_1 \lambda^2 + C_2 \lambda - C_3 = 0 \rightarrow \textcircled{1}$$
$$C_1 = 2 + 1 - 3 = 0 \Rightarrow \boxed{C_1 = 0}$$



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$$C_2 = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ -7 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (-3-2) + (-6) + (2-4)$$

$$= -5-6-2$$

$$C_2 = -13$$

$$C_3 = \begin{vmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{vmatrix} = 2(-3-2) - 2(-6+7)$$

$$= -10-2$$

$$C_3 = -12$$

$$d^3 - 13d + 12 = 0$$

Step 2: To find the eigen values

$$d^3 - 13d + 12 = 0$$

$$d = 1, 3, -4$$

Step 3: To find the eigen vectors

$$(A - dI)X = 0$$

$$\begin{pmatrix} 2-d & 2 & 0 \\ 2 & 1-d & 1 \\ -7 & 2 & -3-d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \textcircled{2}$$

Case (i): $d=1$

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-4}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-4}$$

\therefore The eigen vector is $X_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$



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Case (ii) : $\lambda = -4$

$$\begin{pmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{x_2}{-6} = \frac{x_3}{26}$$

$$\frac{x_1}{1} = \frac{x_2}{-3} = \frac{x_3}{13}$$

\therefore The eigen vector is $X_2 = \begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$

Case (iii) $\lambda = 3$

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

\therefore The eigen vector is $X_3 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

Hence the eigen values & eigen vectors are

λ	1	3	-4
X	$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$



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5) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

Soln:

$$C_1 = 7, C_2 = 0, C_3 = -36$$

$$\lambda^3 - 7\lambda^2 + 36 = 0$$

$$\lambda = -2, 3, 6$$

$$X = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

6) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

Soln:

$$C_1 = 6, C_2 = 11, C_3 = 6$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\lambda = 1, 2, 3$$

$$X = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

7) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$$

Soln:

$$\lambda = 2, 3, 5$$

$$X = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

8) Find the eigen values & eigen vectors of

$$\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$$

Soln:

$$\lambda = 0, 1, 2$$

$$X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$



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Home for

① Find the eigen values and eigen vectors of

(i) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

② Find the eigen values and eigen vectors of

$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = X$

for matrix A eigen values are $\lambda_1, \lambda_2, \lambda_3$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 1$
 $0 = \lambda - 2, \lambda = 2$
 $\lambda = 1$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = X$

$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = X$