UNIT 2 FOURIER SERIES
ROOTMEAN SQUARE VALUE

Root Mean Square (RMS Value) or Effective Value.

Let $f(x)$ be a function defined in an interval (a,b) then $\sqrt{\dfrac{\int_a^b (f(x))^2 dx}{b-a}}$ is called as RMS value of $f(x)$

It is denoted by $\bar{y}$.

$$\bar{y} = \sqrt{\frac{\int_a^b (f(x))^2 dx}{b-a}} \; ; \; \bar{y}^2 = \frac{1}{b-a}\int_a^b (f(x))^2 dx$$

1. Find the RMS value of $f(x) = x - x^2$ in $-1 < x < 1$

$$\bar{y}^2 = \frac{1}{b-a}\int_a^b (f(x))^2 dx$$

$$= \frac{1}{2}\int_{-1}^{1} (x-x^2)^2 dx = \frac{1}{2}\int_{-1}^{1} (x^2+x^4-2x^3) dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^5}{5} - \frac{2x^4}{4}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{5} - \frac{1}{2} - \left[\frac{-1}{3} - \frac{1}{5} - \frac{1}{2}\right]\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{5} - \cancel{\frac{1}{2}} + \frac{1}{3} + \frac{1}{5} + \cancel{\frac{1}{2}}\right]$$

$$= \frac{1}{2}\left[\frac{2}{3} + \frac{2}{5}\right] = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

$$\bar{y} = \sqrt{\frac{8}{15}}$$