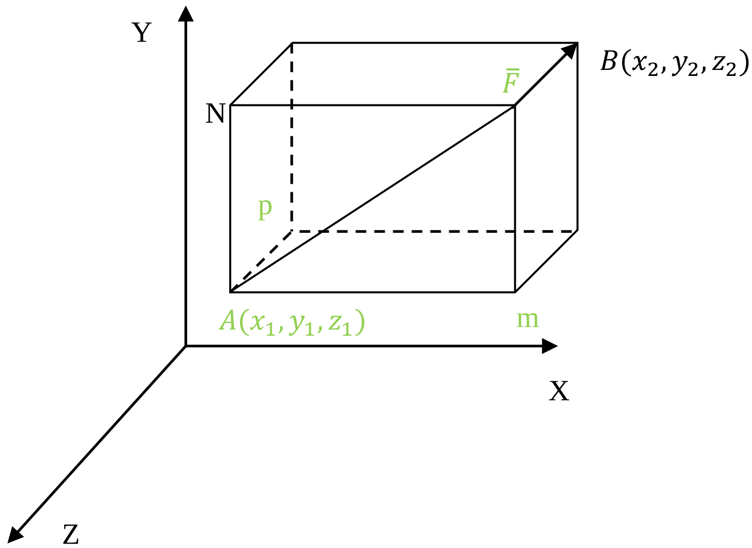
Force vector in co-ordinates




Consider the vector $\overrightarrow{AB}$, joining A and B of co-ordinates ($x_1$, $y_1$, $z_1$) and ($x_2$, $y_2$, $z_2$) respectively.

Let d = Distance between A and B

Now this distance can be resolved into three components along the three co-ordinate axes are dx, dy and dz (which are equal to Am, AN, and Ap)

Distances interms of co-ordinates

$$dx = x_2 - x_1; dy = y_2 - y_1; dz = z_2 - z_1$$

$$\therefore \text{ vector } \vec{AB} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

because $\vec{AB} = dxi + dyj + dzk$

The distance d interms of co-ordinates

$$d = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Unit vector along

$\overrightarrow{AB}$

$$\lambda_{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{dxi + dyj + dzk}{d}$$

We know, $\vec{F}$ is equal to the product of its magnitude and the respective unit vector

$$\vec{F} = F.\alpha_{AB}$$

$$= F.\left(\frac{dxi + dyj + dzk}{d}\right)$$

Scalar components

$$\vec{F} = F_x i + F_y j + F_z k$$

$$F_x = \frac{Fdx}{d} = \frac{F}{d}(x_2 - x_1)$$

III$^{ly}$

$$F_y = \frac{Fdy}{d} = \frac{F}{d}(y_2 - y_1); F_z = \frac{Fdz}{d} = \frac{F}{d}(z_2 - z_1)$$

Angle of inclination with co-ordinate axes

We know $F_x = F\cos\theta_x$ *and* $F_x = \frac{Fdx}{d}$

$$F\cos\theta_x = \frac{Fd_x}{d}$$

$$\theta_x = \cos^{-1}\left(\frac{dx}{d}\right) = \cos^{-1}\left(\frac{x_2 - x_1}{d}\right)$$

III$^{ly}$

$$\theta_y = \cos^{-1}\left(\frac{dy}{d}\right) = \cos^{-1}\left(\frac{y_2 - y_1}{d}\right)$$

$$\theta_z = \cos^{-1}\left(\frac{dz}{d}\right) = \cos^{-1}\left(\frac{z_2 - z_1}{d}\right)$$

Problem 29: A force vector of magnitude 100N, is represented by a line AB of co-ordinates A (1,2,3) and B (5,8,12). Determine

(i) The components of the force along x, y and z axes
(ii) Angles with x, y and z axes

(iii) Specify the force vector.

Solution:

Let the distance of AB=d

The components of distance d, along the co-ordinate axes

$$d_x = x_2 - x_1 = 5 - 1 = 4$$
$$d_y = y_2 - y_1 = 8 - 2 - 6$$
$$d_z = z_2 - z_1 = 12 - 3 = 9$$

$\therefore$ The distance of $AB = d = \sqrt{dx^2 + dy^2 + dz^2}$

$$= \sqrt{4^2 + 6^2 + 9^2} \quad = 11.53$$

(i) Components of the force along the axes

Using $F_x = \frac{Fdx}{d} = \frac{100\times4}{11.53} = 34.69N$

111y $F_y = \frac{Fdy}{d} = \frac{100\times6}{11.53} = 52.03N$

$$F_z = \frac{Fdz}{d} = \frac{100 \times 9}{11.53} = 78.05N$$

(ii) Angles with x, y and z axes

Using $\theta_x = \cos^{-1}\left(\frac{dx}{d}\right) = \cos^{-1}\left(\frac{4}{11.53}\right) = 69.7^0)$

$$\theta_y = \cos^{-1}\left(\frac{dy}{d}\right) = \cos^{-1}\left(\frac{6}{11.53}\right) = 58.64^0)$$

$$\theta_z = cos^{-1}\left(\frac{dz}{d}\right) = cos^{-1}\left(\frac{9}{11.53}\right) = 38.68^0)$$

To check

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2 69.7 + \cos^2 58.64 + cos^2 38.68 = 1$$

$$0.120 + 0.270 + 0.609 = 1$$

$$0.991 \approx 1$$

(iii) Force vector

The force vector $\vec{F}$ is given by

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\therefore \vec{F} = 34.69i + 52.03j + 78.05k$$

Resultant force for a force system in space

Let $\vec{R}$ be the resultant force, writing $\vec{R}$ in Cartesian co-ordinates, we get

$$\overrightarrow{R} = R_x i + R_y j + R_z k$$

Magnitude of resultant force R = $\sqrt{(R_x)^2 + (R_y)^2} + (R_z)^2$

Where $R_x$, $R_y$ and $R_z$ are the scalar components of Resultant force along x, y and z axes respectively.

Direction of resultant force

Let $\theta_{x,}$ $\theta_y$ and $\theta_z$ be the angle of resultant force with x, y and z axes respectively

111y $\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$

$$\theta_Y = \cos^{-1}\left(\frac{R_y}{R}\right)$$

$$\theta_Z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

Resultant force of concurrent spatial forces

The resultant of concurrent spatial forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ etc., acting on a particle is given by

$$\vec{R} = F_1 + F_2 + \cdots$$

$$= (F_{1x} i + F_{1x} j + F_{1z} k) + (F_{2x} i + F_{2y} j + F_{3z} k) + \cdots$$

$$\vec{R} = (\sum F_x) i + (\sum F_y) j + (\sum F_z) k$$

$$\vec{R} = R_x i + R_y j + R_z k$$

$$\therefore Rx = \sum F_x ;\ Ry = \sum F_y ;\ Rz = \sum F_z$$

Equilibrium of particles in space

When a particle is subjected to concurrent force in space, for equilibrium condition resultant force $\vec{R} = 0$

$$R_x i + R_y j + R_z k = 0$$

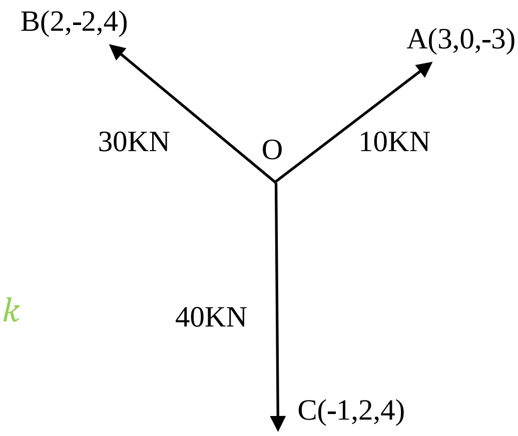
$$(\sum F_x)i + (\sum F_y)j + (\sum F_z)k = 0$$

$$\therefore \sum F_x = 0; \sum F_y = 0; \sum F_z = 0$$

Problem 30:

The lines of action of three forces are concurrent at the origin ‘O’, passes through point A, B and c having coordinates (3,0,-3), (2,-2,4) and (-1,2,4) respectively. If the magnitude of the forces are 10N,30N and 40N, find the magnitude and direction of their resultant.

Solution:




$F_A$ = 10N, $F_B$ = 30N & $F_c$ = 40N

Consider the force passing through A

Position vector

$$\overrightarrow{r_{OA}} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

$$\overrightarrow{r_{OA}} = 3i - 3k$$

Unit vector along OA, $\lambda_{OA} = \frac{\overrightarrow{r_{OA}}}{r}$

$$\lambda_{AB} = \frac{3i - 3k}{\sqrt{3^2 + 3^2}} = \frac{3i - 3k}{4.24}$$

$$\overrightarrow{F_A} = F_A.\lambda_{OA} = 10\left[\frac{3i-3k}{4-2k}\right] = 7.07i - 7.07k$$

Consider the force passing through B

Position vector $\overrightarrow{r_{OB}} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

= 2i – 2 j +4k

Unit vector along OB, $\lambda_{OB} = \frac{\overrightarrow{r_{OB}}}{r} \frac{2i - 2j + 4k}{\sqrt{2^2 + 2^2 + 4^2}} = \frac{2i - 2j + 4k}{4.899}$

$$\overline{F_B} = F_B \lambda_{OB}$$

$$= 30 \left[ \frac{2i - 2j + 4k}{4.899} \right] = 12.247i - 12.247j + 24.49k$$

Consider the force through C

Position vector $\overline{r_{OC}} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)\,k$

$$= -i + 2j + 4k$$

Unit vector along OC,

$$\lambda_{oc} = \frac{\overline{r_{oc}}}{r} = \frac{-i + 2j + 4k}{\sqrt{1^2 + 2^2 + 4^2}} = \frac{-i + 2j + 4k}{4.582}$$

$$\therefore \overline{F_C} = F_C . \lambda_{OC}$$

$$= 40 \left[ \frac{-i + 2j + 4k}{4.582} \right] = -8.729i + 17.459j + 34.919k$$

Resultant force

The resultant force vector is obtained by adding force vector $F_A, F_B \ and \ F_C$

$$\therefore resultant\ force\ \bar{R} = \overline{F_A} + \overline{F_B} + \overline{F_C}$$

i.e $\bar{R} = (7.07i - 7.07k) + (12.247i - 12.247j + 24.49k) + (-8729i + 17.459j + 34.919k)$

$$\bar{R} = 10.59i + 5.212j + 52.343k$$

Magnitude of resultant force

$$R = \sqrt{10.59^2 + 5.212^2 + 52.343^2} = 53.657N$$

Direction of Resultant force

Let the angler of resultant force with x,y and z axes are $\theta_x$, $\theta_y$, $\theta_z$ respectively

Using $\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{10.59}{53.657}\right) = 78.61^0$

lll$^{ly}$ $\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{5.212}{53.657}\right) = 84.42^0$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{52.343}{53.657}\right) = 12.7^0$$

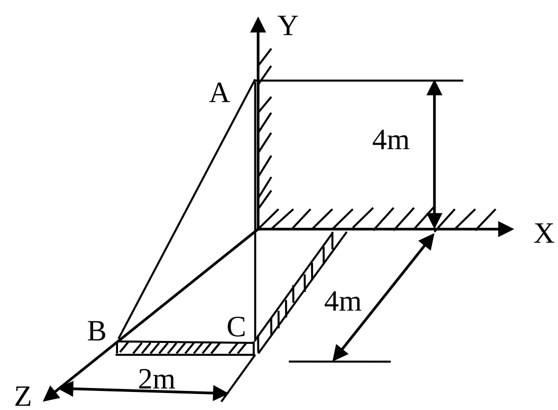
To check

$$cos^2\theta_x + cos^2\theta_y + cos^2\theta_z = 1$$

$$\cos^2 78.61 + \cos^2 84.42 + \cos^2 12.7 \cong 1 \; o.k$$

Problem 31: The tension in cables AB and AC are 100N and 120N respectively in fig. Determine the magnitude of the resultant force acting at A.

Solution:




Considering the tension in cable AB

The force is directed from A to B.

A (0, 4, 0), B (0, 0, 4)

$x_1, y_1, z_1,$ $x_2, y_2, z_2$

A, B – Coordinates

Position vector $\overline{r_{AB}} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1) k$

$$= -4j + 4k$$

$\therefore$ Unit vector along $\overline{AB} = \frac{\overline{r_{AB}}}{r} = \frac{-4j+4k}{\sqrt{4^2+4^2}} = \frac{-4j+4k}{5.656}$

Tension in cable

$$\overline{T_{AB}} = T_{AB}.\lambda_{AB}$$

$$= 100\left[\frac{-4j+4k}{5.656}\right]$$

$$\overline{T_{AB}} = -70.72j + 70.72k$$

Considering tension in cable AC

Now the force is dissected from A to C

A (0, 4, 0) B (2, 0, 4)

$x_1, y_1, z_1$ $x_2, y_2, z_2$

A, B – Coordinates

∴ Position vector $\overline{r_{AC}} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1) k$

$$= 2i - 4j + 4k$$

Unit vector along $\overline{AC}, \lambda_{AC} = \frac{\overline{r_{AC}}}{r} = \frac{2i - 4j + 4k}{\sqrt{2^2 + 4^2 + 4^2}}$

$$= \frac{2i - 4j + 4k}{6}$$

∴ Tension in cable $\overline{AC}, \overline{T_{AC}} = T_{AC} . \lambda_{AC}$

$$= 120 \left[\frac{2i - 4j + 4k}{6}\right]$$

$$\overline{T_{AC}} = 40i - 80j + 80k$$

<u>Resultant force</u>

$$\bar{R} = \overline{T_{AB}} + \overline{T_{AC}}$$

$$= [-70.72j + 70.72k] + [40i + 80j + 80k]$$

Now i, j and k components

$$\bar{R} = 40i - 150.72j + 150.72k$$

∴ Magnitude of Resultant force R = $\sqrt{40^2 + (-150.72)^2 + (150. - 72)^2}$

R = 216.87N