The analysis of heat conduction on extended surfaces (like fins) using the **finite difference method (FDM)** is a numerical approach to solve the heat conduction equation. In extended surfaces, heat is conducted through the material and also convected to the surrounding environment. This scenario can be modeled by a 1D or 2D heat conduction equation, depending on the complexity of the surface.

Governing Equation for Heat Conduction on Extended Surfaces (1D Fin)

The heat conduction equation for a fin is often simplified to the 1D steady-state form, considering uniform cross-sectional area and heat conduction in the x-direction only:

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

Where:

- $T(x)$ = Temperature distribution along the fin
- k = Thermal conductivity of the material
- A = Cross-sectional area
- P = Perimeter of the fin
- h = Convective heat transfer coefficient
- T_∞ = Ambient temperature

For simplification, the governing equation can often be reduced to:

$$\frac{d^2T}{dx^2} - m^2(T - T_\infty) = 0$$

where $m^2 = \frac{hP}{kA}$.

Finite Difference Method (FDM) Approach

To solve this equation numerically, the FDM can be used. In the FDM approach, the continuous domain is discretized into small nodes or points, and the second-order differential terms are approximated using finite difference approximations. For instance, the second derivative $\frac{d^2T}{dx^2}$ at node i is approximated as:

$$\frac{d^2T}{dx^2} \approx \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$

Here, Δx is the spacing between the nodes.

Steps for FDM Solution:

1. **Discretize the domain:** Divide the fin into small segments of equal length Δx, with nodes at each segment boundary.
2. **Set boundary conditions:** These can be:
   - **At the base** (x = 0): The temperature is typically known $T(0) = T_b$.
   - **At the tip** (x = L): Depending on the fin type, there could be different boundary conditions (e.g., insulated tip $\frac{dT}{dx} = 0$ or convective boundary condition).
3. **Set up difference equations:** For each internal node, replace the continuous differential equations with their finite difference counterparts.
4. **Solve the system of equations:** This leads to a system of algebraic equations that can be solved using methods like Gauss elimination, iterative solvers, etc.