UNIT 5 Z - Transforms and Difference equations
Solution of Difference Equation

3. Solve the difference eqn using z-transform
$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$.

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$$

$$z[y_{n+2}] + 6z[y_{n+1}] + 9z[y_n] = z[2^n]$$

$$z^2F(z) - z^2y_0 - zy_1 + 6[zF(z) - zy_0] + 9F(z) = \frac{z}{z-2}$$

$$z^2F(z) + 6z + 9F(z) = \frac{z}{z-2}$$

$$(z^2+6z+9)F(z) = \frac{z}{z-2}$$

$$F(z) = \frac{z}{(z-2)(z^2+6z+9)}$$

$$= \frac{z}{(z-2)(z+3)^2}$$

$$\frac{F(z)}{z} = \frac{1}{(z-2)(z+3)^2} \rightarrow (1)$$

$$= \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$= \frac{A(z+3)^2 + B(z+3)(z-2) + C(z-2)}{(z-2)(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

When $z = 2 \Rightarrow 1 = A(2+3)^2 \Rightarrow 1 = A(25)$ $A = 1/25$

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Solution of Difference Equation

When $z = -3$

$\Rightarrow 1 = C(-3-2)$

$C = -1/5$

When $z = 0$.

$1 = 9A + (-6B) - 2C$

$1 = 9A(1/25) - 6B - 2(-1/5)$

$1 = \frac{9}{25} - 6B + \frac{2}{5}$

$6B = \frac{9+10-25}{25} = \frac{-6}{25}$

$B = \frac{-1}{25}$

① $\Rightarrow \frac{F(z)}{z} = \frac{1/25}{z-2} + \frac{-1/25}{z+3} + \frac{-1/5}{(z+3)^2}$

$F(z) = \frac{1}{25}\left(\frac{z}{z-2}\right) - \frac{1}{25}\left(\frac{z}{z+3}\right) - \frac{1}{5}\left(\frac{z}{(z+3)^2}\right)$

$z^{-1}(F(z)) = \frac{1}{25} z^{-1}\left(\frac{z}{z-2}\right) - \frac{1}{25} z^{-1}\left(\frac{z}{z+3}\right) - \frac{1}{5} z^{-1}\left(\frac{z}{(z+3)^2}\right)$

$= \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n - \frac{1}{5} n(-3)^{n-1}$

$= \frac{1}{25}(2)^n - \frac{1}{25}(-3)^n + \frac{1}{15} n(-3)^n$

UNIT 5 Z - Transforms and Difference equations
Solution of Difference Equation

Solve $y(n+2) - 4y(n+1) + 4(y(n)) = 0$ with $y(0) = 1, y(1) = 0$

using z- transform

$y(n+2) \leadsto z^2 y$

$$y_{n+2} - 4y_{n+1} + 4y_n = 0.$$

$$z[y_{n+2}] - 4z[y_{n+1}] + 4z[y_n] = 0.$$

$$z^2F(z) - z^2y(0) - zy(1) - 4[zF(z) - zy(0)] + 4F(z) = 0$$

$$z^2F(z) - z^2 - 0 - 4[zF(z) - z] + 4F(z) = 0$$

$$z^2F(z) - 4zF(z) + 4F(z) - z^2 + 4z = 0$$

$$(z^2 - 4z + 4)F(z) = z^2 - 4z$$

$$F(z) = \frac{z^2 - 4z}{z^2 - 4z + 4}$$

$$= \frac{z^2 - 4z}{(z-2)^2}$$

$$\frac{F(z)}{z} = \frac{z-4}{(z-2)^2} \rightarrow (1)$$

$$\frac{z-4}{(z-2)^2} = \frac{A}{z-2} + \frac{B}{(z-2)^2}$$

$$= \frac{A(z-2)^{2-1} + B}{(z-2)^2}$$

$$z - 4 = A(z-2) + B$$

When $z = 2$

$$2 - 4 = A(2-2) + B \Rightarrow \boxed{B = -2}$$

$z = 0$

$$0 - 4 = A(0-2) + B$$

$$-4 = -2A - 2 \qquad -2A = -2 \Rightarrow \boxed{A = 1}$$

① $\Rightarrow \quad \frac{F(z)}{z} = \frac{1}{z-2} + \frac{-2}{(z-2)^2}$

$$F(z) = \frac{z}{z-2} - 2\frac{z}{(z-2)^2}$$

$$z^{-1}[F(z)] = z^{-1}\left[\frac{z}{z-2}\right] - 2z^{-1}\left[\frac{z}{(z-2)^2}\right]$$

$$= (2)^n - 2n2^{n-1}$$

$$= 2^n - n2^n$$

$$= 2^n(1-n)$$

5) Solve $y_{n+3} - 3y_{n+1} + 2y_n = 0$. with $y(0)=4, y(1)=0$ and $y(2)=8$.

$$y_{n+3} - 3y_{n+1} + 2y_n = 0$$

$$z[y_{n+3}] - 3z[y_{n+1}] + 2z[y(n)] = 0.$$

$$z^3F(z) - z^3y(0) - z^2y(1) - zy(2) - 3[zF(z) - zy(0)] + 2F(z) = 0$$

$$z^3F(z) - 4z^3 - 0 - 8z - 3[zF(z) - 4z] + 2F(z) = 0.$$

$$z^3F(z) - 4z^3 - 8z - 3zF(z) + 12z + 2F(z) = 0$$

$$(z^3 - 3z + 2)F(z) - 4z^3 + 4z = 0$$

$$(z^3 - 3z + 2)F(z) = 4z^3 - 4z$$

$$\frac{F(z)}{z} = \frac{4z^2 - 4z}{z^3 - 3z + 2}$$

	1	0	-3	2
1	↓	↓	1	-2
	1	1	-2	0

$\Rightarrow (z-1)$

$z^2 + 2z - 2 = 0.$

$(z+2)(z-1) = 0$

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Solution of Difference Equation

$$= \frac{4z^2-4}{(z-1)^2(z+2)} = \frac{4[z^2-1]}{(z-1)^2(z+2)}$$

$$= \frac{4[(z+1)(z-1)]}{(z-1)^2(z+2)}$$

$$\frac{F(z)}{z} = \frac{4(z+1)}{(z-1)(z+2)} \rightarrow (1)$$

$$\frac{4(z+1)}{(z-1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$= \frac{A(z+2)+B(z-1)}{(z-1)(z+2)}$$

$$4(z+1) = A(z+2)+B(z-1)$$

When $z=1$

$$4(1+1) = A(1+2)$$

$$3A = 8$$

$$\boxed{A = 8/3}$$

When $z=-2$

$$4(-2+1) = B(-2-1)$$

$$-4 = -3B$$

$$\boxed{B = 4/3}$$

$(1) \Rightarrow$

$$\frac{F(z)}{z} = \frac{8/3}{(z-1)} + \frac{4/3}{(z+2)}$$

$$F(z) = \frac{8}{3}\left(\frac{z}{z-1}\right) + \frac{4}{3}\left(\frac{z}{z+2}\right)$$

$$z^{-1}[F(z)] = \frac{8}{3}z^{-1}\left(\frac{z}{z-1}\right) + \frac{4}{3}z^{-1}\left(\frac{z}{z+2}\right)$$

$$= \frac{8}{3}(1) + \frac{4}{3}(-2)^n$$

$$= \frac{8}{3} + \frac{4}{3}(-2)^n.$$