UNIT II

ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

(3) Write the quadratic form for the following matrix (i) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$

Soln :

(i) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$.

$$Q = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{23}x_2x_3 + 2a_{31}x_3x_1$$

$$Q = x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$$

(ii) $Q = 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$.

Canonical form :

For a real quadratic form $Q = X^TAX$, the canonical form is Y^TDY (or)

$\lambda_1 y_1^2 + \lambda_2 y_2^2 + \cdots + \lambda_n y_n^2$

Problems :

(1) Determine the nature of the following quadratic form $x_1^2 + 2x_2^2$

Soln :

Matrix of the quadratic form

$$= \begin{pmatrix} \text{Coef of } x_1^2 & \frac{1}{2}\text{coef of } x_1x_2 & \frac{1}{2}\text{coef of } x_1x_3 \\ \frac{1}{2}\text{coef of } x_2x_1 & \text{Coef of } x_2^2 & \frac{1}{2}\text{coef of } x_2x_3 \\ \frac{1}{2}\text{coef of } x_3x_1 & \frac{1}{2}\text{coef of } x_3x_2 & \text{Coef of } x_3^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The eigen values are 1, 2, 0

Nature = Positive Semi definite

(2) Find the rank, index, Signature and nature for the following canonical forms :

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

(iii) $3y_2^2 + 15y_3^2$

Soln :

(i) $y_1^2 + 3y_2^2 + 4y_3^2$

Rank = number of non-zero eigen values

= 3

Index = number of positive square terms = 3

Signature = Difference between positive and negative square terms

= 3 − 0 = 3

Nature = positive definite

(ii) $-y_1^2 + y_2^2 + 4y_3^2$

rank = 3

Index = 2

Signature = 2 − 1 = 1

Nature = Indefinite

(iii) $3y_2^2 + 15y_3^2$

$= 0y_1^2 + 3y_2^2 + 15y_3^2$

rank = 2

Index = 2

Signature = 2

Nature = positive Semi definite

(1) $2x^2 + 3y^2 + 2z^2 + 2xy$

$$= \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \end{array}$$

(2) $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} \end{array}$$